



## WP 16-15

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# ADVERTISING AND AGGREGATE CONSUMPTION: A BAYESIAN DSGE ASSESSMENT

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# Advertising and Aggregate Consumption: A Bayesian DSGE Assessment

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June 6, 2016

## Abstract

Aggregate data reveal that in the U.S. advertising absorbs approximately 2% of GDP and has a well-defined pattern over the business cycle, being strongly procyclical and highly volatile. Because the purpose of brand advertising is to foster sales, we ask whether such spending appreciably affects the dynamics of aggregate consumption and, through this avenue, the economic activity. This question is addressed by developing a dynamic general equilibrium model in which households' preferences for differentiated goods depend on the intensity of brand advertising, which is endogenously determined by profit-maximizing firms. Once the model is estimated to match the U.S. economy, it argues that in the long run the presence of advertising raises aggregate consumption, investment, and hours worked, eventually fostering the whole economic activity. We also find that advertising has a relevant impact on fluctuations in consumption and investment over the business cycle. All of the above mentioned effects are proven to depend crucially on the degree of competitiveness of advertising at the firm level.

JEL Classification: E32, D11, J22, M37

Key words: Advertising, Aggregate Consumption, DSGE model, Bayesian estimation, Business Cycle Fluctuations.

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*“... as a matter of fact, the scale of expenditures on advertising varies positively with the general level of economic activity, so that, insofar as the effect of marginal expenditures is positive, advertising itself tends to accentuate the amplitude of economic fluctuations...”*

*Nicholas Kaldor (1950)*

## 1 Introduction

In 2005, firms spent 272 billion dollars on advertising their products in U.S. media, amounting to approximately 1,200 dollars per civilian citizen. The U.S. advertising industry accounts for 2.2% of GDP, absorbs approximately 20% of firms’ budgets for new investments and uses 16% of their corporate profits. Traditionally, the rationale for firms’ spending on advertising has been identified as the positive effect of advertisements on sales. Firms realize that the demand that they face is not an exogenous product of consumers’ preferences; instead, it can be tilted toward their own products through advertisements. The effectiveness of advertising in enhancing demand is not only revealed by firms’ willingness to spend money on it but also supported by a large number of empirical studies.<sup>1</sup> Building on this fact, we ask whether such relationship holds in the aggregate. Because the reason for advertising is to increase consumer demand, as brand advertising increases the sales of single goods, will aggregate advertising enhance aggregate consumption? If so, will it also increase aggregate demand and production? That is, how important are the spillover effects from the advertising sector to economic activity?

To address these questions, we build and estimate a dynamic stochastic general equilibrium model in which we explicitly account for advertising. The usage of a Bayesian DSGE methodology to assess the advertising-consumption relationship is a novel approach in the literature. We believe that this method is a reasonable choice for at least two reasons. First, advertising is not merely a matter of demand and consumption, but it can affect economic activity through various other channels, for instance, by increasing the substitutability among goods and thereby influencing firms’ market power or by reducing consumers’ savings and thereby decreasing future demand. A general equilibrium framework can conveniently cope with all of these effects. Second, a DSGE model incorporates both steady-state equilibrium and aggregate dynamics, thus providing a unified framework to study the short- and long-run effects of advertising in the aggregate. The developed model is a variant of the neoclassical stochastic growth model augmented with government sector, monopolistically competitive product markets, and real rigidities in the form of habit persistence, adjustment costs and variable capital utilization. Advertising is introduced in this framework by assuming that consumers’ preferences are endogenously determined, depending on the distribution of advertising expenditures across firms. In particular, brand advertising aims to raise the marginal utility of the advertised good, and this assumption creates a positive linkage between advertising and sales. In addition, we assume that rivals’ advertising affects a firm’s sales both by attracting new customers to the market (complementary advertising) and by stealing existing customers from competitors (competitive advertising). This feature

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<sup>1</sup>A survey of empirical studies on the advertising-sales relationship can be found in Vakratsas and Ambler (1999) or Bagwell (2007).

captures the classical dichotomy emphasized in the industrial organization (IO) literature: a firm's advertising may exert both positive and negative spillover effects on the demand faced by competitors (e.g., Roberts and Samuelson, 1988; Karray and Martn-Herrn, 2009).

The model is estimated using quarterly data for the U.S. economy over the 1976:I-2006:IV period. The employed database includes a novel series of total advertising expenditures gathered by aggregating firms' spending on advertisements in a number of heterogeneous U.S. media. The construction of this series was made necessary because advertising is not included among the main business cycle indicators. The results of the Bayesian estimation provides evidence in favor of a positive advertising-consumption relationship, and we show that this finding is crucially related to the relative importance of competitive versus complementary advertising at the firm level. In the long run, in addition to a 6.79% increase in consumption, we find that advertising increases hours worked by 10.9%, GDP by 6.43%, and investment by 4.28%. The underlying mechanism operates through a *work and spend* channel: because of advertising, people work more to consume more and the perceived need for additional consumption is triggered by the advertising signals to which people are exposed. We show that this mechanism makes households unambiguously worse off because the overworking effect more than compensates for the expansion in consumption. Furthermore, we find that the average markup increases (1.87%), thereby providing evidence in favor of an anti-competitive effect of advertising in the U.S. economy. In the short run, the spillover effects of advertising are concentrated on consumption and investment. In particular, we find that the volatility of consumption increases by 23.9.1%, whereas that of investment declines by 14%. These results are driven by a short-run inter-temporal mechanism that pushes households to increase current consumption at the expense of savings. The resulting crowding effect on investment dampens the impact of advertising on output and, in general, softens the quantitative implications of the work and spending mechanism on short-run fluctuations.

The existing literature on the effects of advertising in the aggregate economy is largely empirical. Taylor and Weiserbs (1972), Ashley, Granger and Schmalensee (1980), Jacobson and Nicosia (1981), Chowdhury (1994), and Jung and Seldon (1995) estimate reduced-form specifications using aggregate time series to test for the existence of an advertising-consumption relationship based on Granger causality. Although their results are controversial and not conclusive, some evidence has emerged in favor of bi-directional Granger causality between advertising and aggregate consumption in the U.S. economy. In this paper, we depart from this literature because we analyze the advertising-consumption relationship using a structural econometric approach. This methodology has the advantage of overcoming the potential problem of endogeneity affecting reduced-form estimations cited above and additionally allows for a general assessment of the aggregate spillover effects of advertising. Few other papers examine the macroeconomics of advertising from a theoretical perspective. Bisin and Benhabib (2010) analyze the conditions under which neoclassical theory reproduces the so-called *post-modernist* critique of society. Grossmann (2008) analyzes the welfare implications of the complementarity between advertising and R&D expenditures in a quality-ladder model of endogenous growth. Both papers exploit a general equilibrium setup incorporating persuasive advertising *à la* Dixit-Norman as the one used in our paper. Finally, Hall (2008) analyzes the general equilibrium implications of rent-seeking activities, showing that positive productivity shocks affect retailers' revenues from advertising in a way

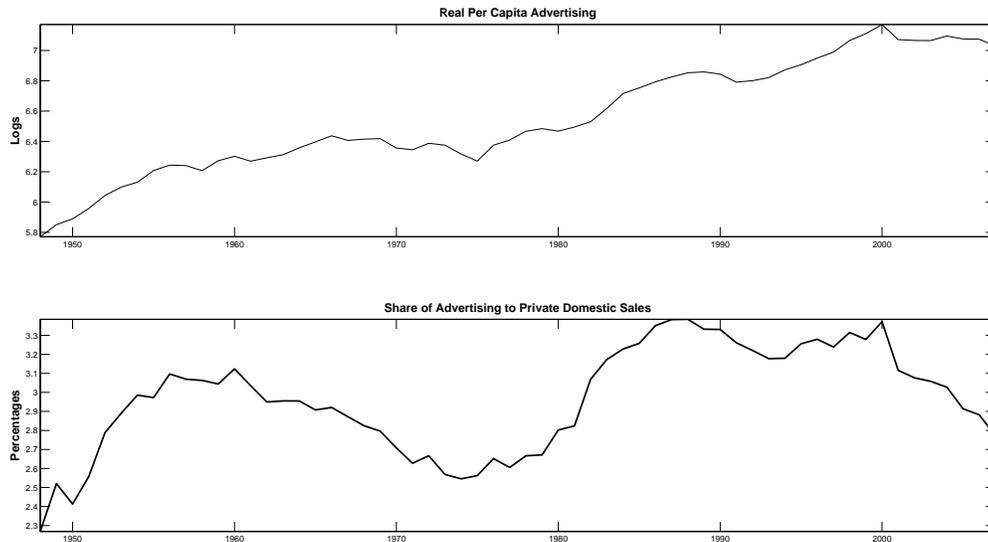


Figure 1: Aggregate Advertising in the U.S.

that makes the latter a procyclical activity. Our results complement this finding, showing that advertising responds in step with output after a variety of shocks regardless of whether they affect productivity.

The remainder of this paper is structured as follows. Section 2 presents the novel quarterly series of aggregate advertising used in the estimation, together with a description of the advertising sector in the U.S. Section 3 provides the DSGE model with advertising, which is estimated in Section 4. Section 5 assesses the short- and long-run effect of advertising on the U.S. economy and evaluates its welfare implications. Section 6 concludes the paper. Details regarding the data and proofs of the propositions are presented in the Appendix.

## 2 Advertising industry in the U.S.

In what follows, we define *aggregate advertising* as the total amount of spending by domestic and foreign firms to advertise their products in domestic media. In Figure 1 we use the annual series of aggregate advertising constructed by Robert J. Coen of Universal McCann to assess the magnitude of aggregate advertising in the U.S. economy during the 1948-2007 period.<sup>2</sup> Panel 1 depicts real per-capita advertising, which works as a proxy for the number of advertising messages received by each individual. The statistics shows an average annual growth rate of 2%, thus indicating that the intensity of advertising per consumer has grown steadily over time. Panel 2 portrays the ratio of aggregate advertising to private domestic sales, which measures the fraction of their resources that firms employ

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<sup>2</sup>Advertising experts consider this to be the most reliable and complete source of data on aggregate advertising. The interested reader can find a full description of Coen's data on Douglas Galbi's blog: [purplemotes.net/2009/05/10/robert-j-coen-advertising-data-hero/](http://purplemotes.net/2009/05/10/robert-j-coen-advertising-data-hero/)

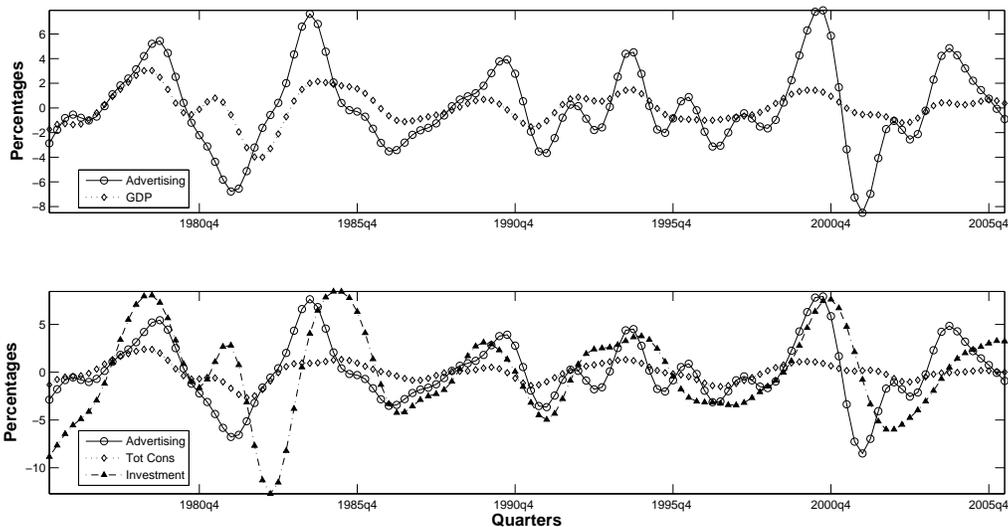


Figure 2: Cyclical Properties of Advertising

in marketing practice. Advertising fluctuates around a long-run mean of 3%, accounting for approximately 20% of investments budgets and absorbing almost the same resources employed in R&D.<sup>3</sup>

What remains unclear from previous analysis is the relationship between advertising and the economic activity in the short run. Although the ratio of advertising to private domestic sales shows large fluctuations and appears positively correlated with GDP, annual data are not an ideal candidate to assess short-run co-movements because they mask information on in-year contractions and expansions. Because quarterly data on aggregate advertising are not available among the standard business cycle indicators,<sup>4</sup> to properly analyze its business cycle properties we construct a novel quarterly series of aggregate advertising. The series exploits two heterogeneous sources of data. First, the quarterly collection of *AD\$Summary* journal, which reports expenditures on media advertisements of the first 1000 firms ranked according to their advertising spending. This source comprises advertising spending in Magazines, Sunday Magazines, Newspapers, Network Televisions, Spot/Local Televisions, Syndicated Televisions, Cable Televisions, Spot/Local Radio, Syndicated Radio, Outdoor. Because advertising spending in newspapers is included only from 1987, we complement *AD\$Summary* data using newspapers revenues from advertising, which are reported at quarterly frequency by the Newspaper Association of America. The resulting series reports the unweighed sum of nominal quarterly expenditures on advertisements in 10 medias covering the period 1976 – 2006. To obtain a real series nominal data are deflated using the GDP deflator, which has been shown to have the closest resemblance to an advertising-spending-specific deflator (see Seldom and Jung, 1995, for further details).

<sup>3</sup>Statistics on R&D in the U.S. are available from the National Science Foundation website ([www.nsf.org](http://www.nsf.org)).

<sup>4</sup>The U.S. federal administration used to collect data on advertising among the indicators used to analyze the business cycle but stopped collecting these data in 1968. Federal data were used by Blank (1962).

Table 1: Real Business Cycle Statistics

$X_t$	$\frac{\sigma(X_t)}{\sigma(Gdp_t)}$	$corr(X_t, Adv_t)$	$corr(X_t, GDP_t)$	$corr(X_t, X_{t-1})$
Advertising	2.30	1	0.71	0.89
GPD	1	0.71	1	0.93
Consumption	0.63	0.66	0.91	0.94
Non-Dur.	0.54	0.62	0.80	0.93
Durables	2.06	0.62	0.88	0.93
Investment	3.00	0.58	0.83	0.93
Hours worked	0.66	0.61	0.90	0.92
Adv/GDP	1.74	0.91	0.36	0.87

Cyclical components have been extracted from data by using the Band Pass filter with (6-32) bands. The series of aggregate advertising has been pre-treated for seasonality using the X11 filter. The series of GDP is GDP minus net export. See Appendix A. for a description of data and sources.

To verify the representativeness of the novel series, we compute the cumulative yearly expenditures from our data and compare them on a year-by-year basis with Coen’s data.<sup>5</sup> Over the sample, our series account on average for 80% of Coen’s data and adequately tracks the composition of spending across media. Specifically, advertising spending in Televisions accounts for 38.5% of total aggregate advertising in our data vs 37.9% in Coen’s, Newspapers 43.5% vs 36.7%, Outdoors 1.7% vs 2.4%, Magazines 14% vs 11.5%, Radio 2.2% vs 11.5%. Radio appears the only severe case of bias, which is most likely due to own nature of radio advertisements, typically paid by small, local firms that are not included in the sample of large advertisers gathered by AD\$Summary. It is worth noting that the importance of replicating the composition of spending across media is given by the substantial difference in the cyclical properties of advertising across media. In this perspective, the construction of a multi-media series improves upon the usage of single media spending to proxy for aggregate advertising because it reduces the potential bias in measuring cyclical properties.<sup>6</sup>

In Figure 2, we plot the cyclical component of real advertising expenditures along with real GDP (panel 1), real total consumption and real fixed private investment (panel 2). Advertising appears procyclical, more volatile than GDP, consumption and hours worked, and less volatile than investment. Table 1 reports the related business cycle statistics, which confirm these findings. Advertising displays a positive correlation with GDP (0.71) and is more than twice as volatile as GDP. Also, it is persistent over the cycle, with a point estimate of the first-order autocorrelation of 0.89.

Concerning the other macroeconomic aggregates, advertising displays the strongest cor-

<sup>5</sup>Because Coen’s data include spending in some media that are not reported in AD\$Summary, we subtracted the corresponding expenditures from his grandtotal before the comparison.

<sup>6</sup>Further details on this issue and on the distribution of spending across media are available in the Technical Appendix. Regarding other aspects of the data, our aggregate series do not provide information on advertising at the firm or sector level, thus not allowing for further analyses on the composition of expenditures across industries, or on the distribution of advertisers (e.g. according to their size, or location).

Table 2: Dynamic cross-correlations

$X_t$	<b>k</b>								
	<b>-4</b>	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<i>corr</i> ( $X_t, GDP_{t+k}$ )								
Advertising	0.06	0.30	0.51	0.66	0.71	0.66	0.54	0.39	0.26
Consumption	0.21	0.42	0.63	0.81	0.91	0.91	0.79	0.59	0.35
Investment	0.53	0.76	0.90	0.93	0.83	0.65	0.40	0.15	-0.08
	<i>corr</i> ( $X_t, ADV_{t+k}$ )								
Consumption	0.25	0.35	0.46	0.56	0.66	0.66	0.58	0.42	0.22
Non-Dur.	0.27	0.40	0.51	0.59	0.62	0.58	0.46	0.30	0.13
Durables	0.08	0.18	0.31	0.47	0.62	0.71	0.69	0.57	0.36
Investment	0.53	0.66	0.72	0.70	0.58	0.40	0.18	-0.05	-0.26
Hours worked	0.33	0.45	0.56	0.62	0.61	0.50	0.31	0.08	-0.15

relation with total consumption (0.66). Moreover, it is more than 3 times more volatile than total consumption, almost 4 times more volatile than non-durable consumption, slightly more volatile than durable consumption and 30% less volatile than investment. We also find that the ratio of advertising to GDP is positively correlated with GDP itself. This result indicates that firms do not spend a constant proportion of their revenues on advertising, as predicted by the standard theory of optimal advertising budgeting (e.g. Dorfman and Steiner, 1954).

Finally, we analyze the dynamic cross-correlations between advertising, GDP, consumption, hours worked and investment. Dynamic cross-correlations provide useful evidence to support or dismiss the common idea that advertising is a leading indicator of the cycle (see Blank, 1962). According to Table 2, advertising does not lead GDP – the highest correlation between the two variables is contemporaneous (0.71 at  $k = 0$ ) and the distribution looks symmetric around 0 up to the third order lead/lag – and is contemporaneously correlated with total consumption – the strongest correlations are at  $k = 0$  and  $k = 1$  (0.66). It lags durable goods – the two highest correlations are at  $k = 1$  (0.71) at  $k = 2$  (0.69) – and strongly leads investment – the two highest correlations are at  $k = -1$  (0.70) at  $k = -2$  (0.72). Overall, the dynamic cross-correlation analysis denies the role of advertising as leading indicator of the cycle.<sup>7</sup>

<sup>7</sup>We tested the robustness of the findings presented in this section along several dimensions: using different data, filters, definitions of aggregate advertising, and advertising to output ratios. The main conclusions are confirmed in all of the considered cases. Aggregate advertising is always highly volatile, procyclical, persistent, and never leads output; and its ratio with different measures of output always appears procyclical.

### 3 A DSGE model with advertising

We consider an economy in which a continuum of differentiated goods indexed by  $i \in [0, 1]$  are produced by monopolistically competitive producers. Goods are sold by firms to households for consumption and investment purposes and to the government, which collects taxes from households to finance public expenditures. Households' preferences for differentiated goods are not exogenous but depend on the distribution of advertising expenditures across firms. Firms are aware of this linkage and compete in the market jointly using advertising budgeting and pricing policy. All of the interactions among firms, households and the government occur in a stochastic environment in which short-run dynamics are driven by several demand and supply shocks.<sup>8</sup>

#### 3.1 The representative household and the role of advertising

The representative household has preferences in period 0 given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[ \left( \tilde{C}_t - \zeta \tilde{C}_{t-1} \right) / \Gamma_t \right]^{(1-\sigma)} - 1}{1-\sigma} - \xi_t^h \frac{H_t^{1+\phi}}{1+\phi} \right\} \quad (1)$$

where  $\tilde{C}_t$  is a consumption aggregate,  $H_t$  is hours worked,  $E_t$  denotes the mathematical expectations operator conditional on the information available at time  $t$ ,  $\beta \in (0, 1)$  represents the subjective discount factor,  $\zeta \in [0, 1]$  controls for the degree of internal habit persistence, and  $\{\phi, \sigma\} \in R_+^2$  are preference parameters.  $\xi_t^h$  is a preference shock that follows a univariate autoregressive process of the form  $\log(\xi_t^h/\xi) = \rho_h \log(\xi_{t-1}^h/\xi) + \epsilon_t^h$ , in which  $\rho_h \in [0, 1)$ ,  $\xi > 0$  and  $\epsilon_t^h$  are i.i.d. innovations with a mean 0 and standard deviation  $\sigma_h$ . As in An and Schorfheide (2007), we assume that households derive utility from the object  $\tilde{C}_t - \zeta \tilde{C}_{t-1}$  relative to the deterministic level of labor augmenting technological progress,  $\Gamma_t$ , which evolves over time at the constant rate  $\tau > 0$ . This assumption is necessary to ensure that the economy evolves along a balanced growth path.

Building on Dixit and Norman (1978), we assume that the composite consumption aggregate,  $\tilde{C}_t$ , takes the following form:

$$\tilde{C}_t = \left( \int_0^1 (c_{i,t} - B(f_{i,t}, f_{-i,t}, \Gamma_t))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2)$$

where  $c_{i,t}$  denotes the quantity of good  $i$  consumed in period  $t$ ,  $f_{i,t}$  and  $f_{-i,t}$  respectively summarize the effects of a firm's own and its competitors' advertising outlays on preferences, and  $\varepsilon > 1$  is the pseudo-elasticity of substitution across varieties. The influence of advertising on consumers' brand choices is captured by the additive term  $B(f_{i,t}, f_{-i,t}, \Gamma_t)$ , which is

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<sup>8</sup>This section describes the main features of the model, while the complete set of model equations can be found in Appendix B.

assumed to be increasing and concave in its first argument and decreasing and convex in its second argument, satisfying  $B(0, 0, \Gamma_t) = 0$  for any  $\Gamma_t$ .<sup>9</sup>

The formulation adopted implies that advertising is *persuasive* in that it affects consumers' choices by modifying their tastes. Among the main competing views of advertising proposed in the literature - persuasive, informative, and complementary (see Bagwell, 2007, section 2.2) - we rely on persuasive advertising because the marketing literature has provided convincing empirical evidence in support of this view (e.g. Stern and Resnik, 1991; Vakratsas and Ambler, 1999; Kamakura and Russell, 1993; McClure et al., 2004). In the model, the persuasive effect is replicated by assuming that exposure to brand advertising creates a negative externality that induces households to feel dissatisfaction with their current levels of consumption. As a result, the marginal utility of advertised goods increases, and households devote larger fractions of their income to purchasing advertised products. The consumption aggregate (2) also implies that brand advertising decreases the substitutability between advertised goods and rival brands. As documented in several empirical studies, building brand equity thorough advertising is a successful strategy to differentiate firms' own products from rival brands (Kamakura and Russell, 1993), either because advertising improves the perceived product quality (Aaker and Jacobson, 1994) or because it reinforces customers' loyalty (Yoo, Donthu and Lee, 2000). In the theoretical literature, this characteristic is typically modeled by assuming that advertising affects the price elasticity of demand (e.g., Dixit and Norman, 1978), which in our formulation is guaranteed by the non-homotheticity of the consumption aggregate (2).

The representative household determines the optimal demand for consumption goods by minimizing consumption expenditures subject to the consumption aggregate (2), i.e.,

$$c_{i,t} = (p_{i,t}/P_t)^{-\varepsilon} \tilde{C}_t + B(f_{i,t}, f_{-i,t}, \Gamma_t) \quad (3)$$

where  $P_t = [\int_0^1 p_{i,t}^{1-\varepsilon} di]^{1/(1-\varepsilon)}$  is the price index. Equation (3) captures several features of the advertising-sales relationship that are often emphasized in the IO and marketing literature and documented by a large body of empirical studies. First, sales react positively to advertising campaigns (Vakratsas and Ambler, 1999; Bagwell, 2007). In the model, an incremental investment in advertising raises  $f_{i,t}$ , which in turn increases  $B(\cdot)$ , thus shifting the demand function (3) to the right. Additionally, the concavity of  $B(\cdot)$  implies that advertising has diminishing marginal returns on the demand in accordance with the empirical evidence reported in Deighton, Henderson and Neslin (1994) and Vakratsas et al. (2004).

Second, in multi-product markets, advertising is typically *competitive* in that rival advertisements negatively affect firms' demand (Karray and Martn-Herrn, 2009). In the model, an expansion in rival advertisements  $f_{-i,t}$  decreases  $B(\cdot)$ , thus shifting the demand function (3) to the left. As noted by Roberts and Samuelson (1988), however, rival advertising may also increase consumers' awareness of the existence of a product category, thus attracting new customers to the market and enhancing the market size. In the model, this feature of advertising – usually referred to as the *complementarity* property – arises as a general equilibrium outcome whenever an increase in aggregate advertising results in a larger amount

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<sup>9</sup>The dependence of function  $B(\cdot)$  on the rate of labor-augmenting technological progress,  $\Gamma_t$ , is a necessary condition to guarantee that a balanced growth path equilibrium exists. See Molinari and Turino (2009) for further details.

of aggregate consumption  $C_t$ . As a matter of fact, because  $P_t C_t = \int_0^1 p_{i,t} c_{i,t} di$ , the optimal demand (3) implies

$$C_t = \tilde{C}_t + \int_0^1 (p_{i,t}/P_t) B(f_{i,t}, f_{-i,t}, \Gamma_t) di \quad (4)$$

which reveals that, for a fixed level of utility  $\tilde{C}_t$ , the (conditional) demand of aggregate consumption is a function of the distribution of advertising. In this respect, the integral term in equation (4) controls for the complementary effect of advertising in the model, determining whether and how changes in advertising expenditures shift the aggregate demand of consumption.

Finally, there is broad consensus in the literature regarding the carryover effect of advertising. Many empirical papers show that past advertising campaigns influence current purchases, thus making advertising an inherently dynamic variable (Clarke, 1976; Leone, 1995). The dynamic characteristic of advertising is replicated in the model by assuming that current and past advertising sum to create a reputation for a good, the producer's *goodwill*, which is defined as the intangible stock of advertising that affects a consumer's tastes at time  $t$ . As in Nerlove and Arrow (1962), we assume that the stock of goodwill evolves according to the law of motion:

$$f_{i,t} = (1 - \delta_f) f_{i,t-1} + (1 - \phi_z(z_{i,t}/z_{i,t-1})) z_{i,t} \quad (5)$$

where  $z_{i,t}$  is firm's investment in new advertising at time  $t$  and  $\delta_f \in (0, 1)$  is the goodwill depreciation rate. It is easy to verify that the law of motion (5) and the demand function (3) jointly imply that both current and past advertising expenditures affect firms' own demand, with an intensity that decays over time. The function  $\phi_z(\cdot)$  captures the presence of convex adjustment costs in advertising and satisfies  $\phi_z(\tau) = \phi'_z(\tau) = 0$  and  $\phi''_z(\tau) > 0$ . This assumption allows for a sluggish response of advertising to exogenous shocks, which may be driven by the costs that firms encounter when adjusting their current level of advertising expenditures, e.g. administrative costs or time to build new advertising campaigns.

For the rest of the demand side of the model, we assume that each household holds one asset, the capital stock  $\bar{K}_t$ , and in each period  $t$  chooses the capital utilization rate,  $u_t$ , to transform physical capital into effective capital according to the rule  $K_t = u_t \bar{K}_t$ . The cost of capital utilization is defined in units of physical capital and is given by  $a(u_t)$ , which is assumed to be increasing and convex in  $u_t$ . Additionally, we assume that in the steady state,  $u_t = 1$  and  $a(1) = 0$ . Physical capital evolves according to the law of motion

$$\bar{K}_{t+1} = (1 - \delta_k) \bar{K}_t + \xi_t^k (1 - \phi_k(I_t/I_{t-1})) I_t \quad (6)$$

where  $I_t$  is investment in new capital,  $\delta_k \in (0, 1)$  is the capital depreciation rate, and  $\phi^k(\cdot)$  denotes convex adjustment costs for capital, satisfying  $\phi_k(\tau) = \phi'_k(\tau) = 0$  and  $\phi''_k(\tau) > 0$ .  $\xi_t^k$  is an investment-specific shock that follows a univariate autoregressive process of the form  $\log(\xi_t^k) = \rho_k \log(\xi_{t-1}^k) + \epsilon_t^k$ , in which  $\rho_k \in [0, 1)$  and  $\epsilon_t^k$  are i.i.d. innovations with a mean equal to 0 and standard deviation  $\sigma_k$ . Investment in new capital is assumed to be a composite good produced by aggregating differentiated goods via technology  $I_t = (\int_0^1 i_{i,t}^{(\epsilon-1)/\epsilon} di)^{\epsilon/(\epsilon-1)}$ . The resulting optimal demand for good  $i$  for investment purposes is not directly affected by advertising. This assumption naturally fits in our setup because investment represents

the avenue through which consumption is postponed until the future, whereas advertising is intended to induce urge in consumption.

Each household supplies labor services per unit of time and rents physical capital to firms. Labor and capital markets are perfectly competitive, with a wage rate  $W_t$  paid per unit of labor services and a rental rate  $R_t^k$  paid per unit of capital. In addition, households receive profit  $\Pi_t$  for the ownership of firms and pay lump-sum taxes  $T_t$  to finance an exogenous and stochastic government spending  $G_t$ . In each period, the government allocates the spending  $G_t$  over a basket of intermediate goods to maximize the amount of the composite good produced with the CES technology  $G_t = (\int_0^1 g_{i,t}^{(\epsilon-1)/\epsilon})^{\epsilon/(\epsilon-1)}$ . As a result, government demand is not affected by advertising, which is a conservative assumption with respect to our results. A positive linkage between advertising and government spending would in fact strengthen the effect of advertising on aggregate demand.

### 3.2 Firms

Firms produce output by combining labor and capital with a Cobb-Douglas technology of the form

$$y_{i,t} = A_t^y k_{i,t}^{1-\alpha} (\Gamma_t h p_{i,t})^\alpha \quad (7)$$

where  $\alpha \in (0, 1)$  and  $y_{i,t}$ ,  $k_{i,t}$ ,  $h p_{i,t}$  respectively denote output, capital stock, and production-related labor.  $A_t^y$  measures a purely transitory technology shock that evolves according to  $\log(A_t^y) = \rho_y \log(A_{t-1}^y) + \epsilon_t^y$ , in which  $\rho_y \in [0, 1)$  and  $\epsilon_t^y$  are i.i.d. innovations with a mean equal to 0 and standard deviation  $\sigma_y$ .

Each firm may promote its product by incurring advertising expenditures. To avoid complications related to the modeling of an advertising-specific sector, we follow Grossmann (2008) by assuming that advertisements are produced in-house using the following technology

$$z_{i,t} = A_t^z \Gamma_t h a_{i,t}^{\alpha_z} \quad (8)$$

where  $\alpha_z \in (0, 1)$  and  $h a_{i,t}$  denotes advertising-related labor.  $A_t^z$  measures advertising-specific productivity and evolves according to  $\log(A_t^z) = \rho_z \log(A_{t-1}^z) + \epsilon_t^z + \rho_{zy} \epsilon_t^y$  in which  $\rho_z \in [0, 1)$ ,  $\rho_{zy} \in [0, 1]$  and  $\epsilon_t^z$  are i.i.d. innovations with a mean equal to 0 and standard deviation  $\sigma_z$ . The dependence of advertising-specific shocks on innovations  $\epsilon_t^y$  is intended to capture movements in advertising spending driven by changes in the overall productivity of the economy. We therefore refer to  $A_t^y$  as an *economy-wide* productivity shock.

Firm  $i$  sells its production  $y_{i,t}$  to meet the demand for consumption, investment and government purchases, i.e.,

$$y_{i,t} = (p_{i,t}/P_t)^{-\epsilon} (\tilde{C}_t + I_t + G_t) + B(f_{i,t}, f_{-i,t}, \Gamma_t) \quad (9)$$

It is easy to verify that the price elasticity of demand,  $\eta_{y,p}(i) = \epsilon(1 - B(\cdot)/y_{i,t})$  is a decreasing function of the goodwill stock. As argued in section 3.1, this property replicates the brand equity effect of advertising and results from the non-homotheticity of the consumption aggregate (2).

Firms jointly determine pricing policy, production plans and advertising budgeting by maximizing the discounted flow of future profits subject to the constraints given by the

demand function (9), the law of motion of the goodwill stock (5), and the technology to produce goods (7) and advertising (8). Optimal planning satisfies two conditions. First, firms set the optimal price by charging a positive markup over the marginal cost, i.e.,

$$p_{i,t} = \eta_{y,p}(i) / [(\eta_{y,p}(i) - 1)] \varphi_t \equiv \mu_{i,t} \varphi_t \quad (10)$$

where  $\varphi_t$  is the marginal cost of production and  $\mu_{i,t}$  is the optimal markup, which is firm specific and increasing in  $g_{i,t}$ , as a result of the brand loyalty effect described above. Under the second optimality condition, the firm spends on advertising until the marginal benefit from an additional unit of advertising equals its marginal cost, as stated in the following Euler equation:

$$(p_{i,t} - \varphi_t) \partial B(\cdot) / \partial f_{i,t} + (1 - \delta_f) E_t \{ \nu_{i,t+1} Q_{t,t+1} \} = \nu_{i,t} \quad (11)$$

where  $Q_{t,t+1}$  is the stochastic discount factor and  $\nu_{i,t}$  is the marginal cost of producing advertising. Consistent with the dynamic nature of goodwill, the marginal benefit that appears in the left-hand side of equation (11) is the one-period-ahead expected payoff from a marginal unit of advertising in addition to the discounted opportunity cost of not producing tomorrow the surviving goodwill produced today. Conditions (10) and (11) jointly imply that advertising and pricing are complementary policies, and therefore, any shock that causes firms to revise their pricing policies will also cause them to adjust advertising spending in the same direction.

### 3.3 The general equilibrium

We restrict the analysis to symmetric equilibria in which all firms set the same price, produce the same quantity of goods, and invest the same amount of resources in advertising. In addition, we normalize the price of goods to 1 so that all of the remaining prices are defined in terms of contemporaneous consumption. In general, the presence of advertising has countervailing effects on macroeconomic aggregates, and in the next section, we employ a numerical analysis to unravel the general equilibrium results. However, in the case of *purely competitive* advertising, i.e. when the complementary effect is switched off, the general equilibrium properties can be unambiguously characterized. The following proposition summarizes these results.

**Proposition 1.** *Consider an economy in which monopolistically competitive firms can promote their products by incurring advertising expenditures. Let the households' intertemporal utility function and the consumption aggregate be of the forms (1) and (2), respectively. Additionally, assume that the technology to produce goods and advertising is given by (7) and (8), respectively, and that capital and labor markets are perfectly competitive. Then, at the stationary perfect foresight symmetric equilibrium, purely competitive advertising reduces the equilibrium levels of consumption, output and investment and increases hours worked with respect to an identical economy in which advertising is banned.*

According to Proposition 1, purely competitive advertising is wasteful in the long run. In this case, advertising becomes a zero-sum game that leaves the demand function of firms' products unaffected in equilibrium. Firms do not internalize this general equilibrium effect

in their optimal decisions and continue hiring workers to produce advertisements. As a result, the labor demand shifts to the right with respect to an identical economy in which advertising is banned. The equilibrium levels of hours worked thus increases, but a fraction of production-related labor is now reallocated to an unproductive activity (advertising). As a consequence, the equilibrium level of output diminishes and this induces a negative income effect that eventually reduces consumption and investment.

## 4 Parameters estimates

### 4.1 Method and data

To estimate the model, we follow the Bayesian paradigm: a prior distribution is chosen for each structural parameter, and a Metropolis-Hastings algorithm then estimates the posterior distribution.<sup>10</sup> The estimation is performed using quarterly U.S. data on real personal consumption expenditures, real output minus net exports, total hours worked, real private nonresidential fixed investment and the series of real aggregate advertising presented in Section 2. All data are in per-capita terms and are expressed as quarterly growth rates with the exception of hours worked, which is expressed as demeaned log-levels. The sample period is from the first quarter of 1976 to the end of 2006, the time span for which quarterly data on aggregate advertising are available. The corresponding vector of measurement equations is defined as follows:

$$\begin{bmatrix} dlCONS_t \\ dlGDP_t \\ lHOURS_t \\ dlADV_t \\ dlINV_t \end{bmatrix} = \begin{bmatrix} \tau^* \\ \tau^* \\ 0 \\ \tau^* \\ \tau^* \end{bmatrix} + \begin{bmatrix} dlC_t \\ dlGDP_t \\ lH_t \\ dlZ_t \\ dlI_t \end{bmatrix}$$

where  $l$  and  $dl$  stand for log and log difference, respectively, and  $\tau^* = \log(\tau)$ .

### 4.2 Functional form assumptions

The model estimation requires the functions controlling for (i) the effectiveness of advertising in preferences, (ii) the cost of capital utilization and (iii) adjustment costs to be parameterized. First, we assume that  $B(\cdot)$  satisfies the following condition:

$$B(\cdot)/\Gamma_t = S(f_{i,t}, \Gamma_t) - \gamma S(f_{-i,t}, \Gamma_t) \text{ with } \gamma \in [0, 1] \quad (12)$$

where

$$S(x_t, \Gamma_t) = \frac{\theta x_t / \Gamma_t}{1 + \theta x_t / \Gamma_t} \text{ with } x_t = \{f_{i,t}; f_{-i,t}\} \quad (13)$$

This formulation conforms to the assumptions made in section 3.1 and guarantees that a balanced growth path equilibrium exists. As in Grossmann (2008), we assume that rival advertising is measured by the average stock of competitors' goodwill:  $f_{-i,t} = \int_0^1 f_{j,t} dj$ . The

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<sup>10</sup>For the estimation, the model is first de-trended and then log-linearized around the steady state equilibrium.

choice of  $S(\cdot)$  implies that without advertising (i.e.  $f_{i,t} = 0 \forall i$  and  $t$ ), the consumption aggregate (2) takes the standard Dixit-Stiglitz form with constant elasticity of substitution across varieties. Parameter  $\theta > 0$  controls for the effect of brand advertising on consumer's preferences, and can be interpreted as a measure of the advertising intensity in enhancing the demand of goods at the firms level.

The specification of function  $B(\cdot)$  nests any intermediate type of advertising between two extreme cases in which advertising either merely redistributes market shares across firms or works as a pure market-enhancing force. The mapping between these two cases is controlled by parameter  $\gamma$ , which measures the degree of competitiveness of advertising at the firm level. Its role is apparent if one examines equation (12). When  $\gamma = 1$ , the effectiveness of brand advertising in enhancing sales of a firm depends on the intensity of rivals' advertising. At the symmetric equilibrium, advertising is a zero-sum game because the negative effect of rival goodwill precisely compensates for the effect of firms' own goodwill, thus leaving the demand function unaffected. We refer to this case as *purely competitive advertising*. Conversely, when  $\gamma = 0$ , the demand-stealing mechanism is switched off (i.e.,  $\partial B(\cdot)/\partial f_{-i,t} = 0 \forall i$ ), and brand advertising expands the producer's sales regardless of the intensity of competitors' advertising. Because of this effect, the advertising of each individual firm jointly exerts upward pressure on the aggregate demand, which eventually spills over to all firms. Finally, when  $\gamma \in (0, 1)$ , the parametrization yields a combination of the two extreme cases in which firms' advertising both steals customers from competitors and expands the size of the market. We refer to this general case as *market-enhancing advertising*.

With regard to adjustment costs, we adopt standard quadratic functions satisfying  $\phi_z''(\tau) = \psi_z > 0$  and  $\phi_k''(\tau) = \psi_k > 0$ . Finally, the cost of capital utilization is parametrized by assuming that  $a(u_t) = R_{ss}^k (\omega u_t^2/2 + (1 - \omega)u_t + \omega/2 - 1)$  where  $R_{ss}^k$  denotes the rental rate of capital evaluated in the steady state and  $\omega > 0$  is a parameter that controls for the curvature of  $a(\cdot)$ . In the estimation of the model, we adopt the normalization used in Iacoviello and Neri (2010) and Smets and Wouters (2007) by specifying the prior distribution for the curvature parameter in terms of  $\psi_u = \omega/(1 + \omega)$ .

### 4.3 Prior distributions

Prior distributions are summarized in Table 3. The elasticity of labor in the production of advertising,  $\alpha_z$ , is beta distributed with a mean of 0.73 and a standard deviation of 0.02. Conditional on the remaining parameters, this prior mean implies a steady-state ratio of advertising-related labor to total hours worked ( $Ha/H$ ) of 0.8%, which corresponds to the average fraction of the labor force employed in the advertising industry during the 2002-2012 period, as reported by the Bureau of Labor Statistics. The prior for  $\gamma$  is a uniform  $[0,1]$  distribution that reflects our neutral stance between competitive and market-enhancing advertising. The goodwill depreciation rate  $\delta_f$  is assumed to follow a quite disperse beta distribution with a mean of 0.15, implying a half-life of the goodwill stock of about four quarters. This value is consistent with the empirical evidence provided in Clarke (1976) and Leone (1995), according to which the effect of advertising on the firm's demand is short-lived. The parameter controlling for advertising adjustment costs,  $\phi_z$ , is assumed to be gamma distributed with a mean of 4 and a standard error of 1. This prior implies an equivalent elasticity of the adjustment costs in the laws of motion of capital

Table 3: Prior and Posterior Distributions

Parameter	Density	<i>Priors</i>			<i>Posteriors</i>	
		Domain	Mean	90% interval	Mean	90% interval
$\beta$	Beta	$[0, 1]$	0.995	[0.991 0.998]	0.993	[0.989 0.997]
$\sigma$	Gamma	$R_+$	2.000	[1.255 2.887]	1.366	[0.857 1.896]
$\phi$	Gamma	$R_+$	0.770	[0.167 1.732]	2.143	[1.150 3.073]
$\gamma$	Uniform	$[0, 1]$	0.500	[0.050 0.950]	0.593	[0.354 0.822]
$\delta_f$	Beta	$[0, 1]$	0.150	[0.025 0.343]	0.164	[0.069 0.279]
$\alpha$	Beta	$[0, 1]$	0.708	[0.674 0.740]	0.739	[0.714 0.770]
$\alpha_z$	Beta	$[0, 1]$	0.734	[0.700 0.766]	0.717	[0.685 0.748]
$\tau^*$	Normal	$R$	0.005	[.0017 .0083]	.0045	[.0040 .0049]
$\psi_k$	Gamma	$R_+$	4.000	[2.509 5.774]	3.141	[1.822 4.350]
$\psi_z$	Gamma	$R_+$	4.000	[2.509 5.774]	1.227	[0.464 1.969]
$\psi_u$	Beta	$(0, 1)$	0.500	[0.171 0.828]	0.360	[0.104 0.602]
$\zeta$	Beta	$[0, 1]$	0.500	[0.171 0.828]	0.321	[0.178 0.467]
$\rho_y$	Beta	$[0, 1]$	0.500	[0.172 0.828]	0.940	[0.892 0.985]
$\rho_h$	Beta	$[0, 1]$	0.900	[0.807 0.967]	0.945	[0.917 0.974]
$\rho_g$	Beta	$[0, 1]$	0.500	[0.172 0.828]	0.956	[0.934 0.978]
$\rho_k$	Beta	$[0, 1]$	0.500	[0.172 0.828]	0.750	[0.606 0.900]
$\rho_z$	Beta	$[0, 1]$	0.900	[0.807 0.967]	0.842	[0.720 0.960]
$\rho_{zy}$	Beta	$[0, 1]$	0.800	[0.505 0.981]	0.804	[0.593 0.998]
$\sigma_y$	InvGamma	$R_+$	0.008	[0.002 0.022]	0.006	[0.005 0.007]
$\sigma_h$	InvGamma	$R_+$	0.006	[0.001 0.017]	0.022	[0.014 0.029]
$\sigma_g$	InvGamma	$R_+$	0.010	[0.002 0.029]	0.048	[0.043 0.053]
$\sigma_k$	InvGamma	$R_+$	0.008	[0.002 0.022]	0.025	[0.015 0.035]
$\sigma_z$	InvGamma	$R_+$	0.006	[0.001 0.017]	0.041	[0.027 0.056]

Note: The algorithm for the Bayesian estimation works as follows. First, the posterior kernel is maximized to find the mode of the posterior distribution. Second, starting from a random perturbation around the mode, a random-walk Metropolis-Hastings algorithm is used to sample from the posterior distribution. We run this algorithm 4 times from different perturbation points, eventually building 4 chains of 120,000 draws each. This strategy seems to ensure a relatively rapid convergence of the Markov chains. The convergence diagnostics indicate that approximately 60,000 draws are sufficient to attain convergence for all of the parameters. Finally, we report selected statistics for the posterior distributions by computing the average of correspondent moments from all of the chains, wherein we discard the initial 40% of observations from each chain to remove the dependence from the initial condition.

and advertising, which reflects our *a priori* lack of knowledge about these adjustment cost functions. Prior distributions for the other parameters are consistent with previous studies (Smets and Wouters, 2007; Iacoviello and Neri, 2010).

Five parameters are fixed in the estimation procedure, either because they are typically difficult to identify in estimated DSGE models or to improve identification of the remaining parameters. The gross elasticity of substitution across varieties,  $\varepsilon$ , and the depreciation rate of capital,  $\delta_k$ , are fixed at 11.27 and 0.010, respectively, to match the long-run average markup (1.11) and investment share (20%). The intensity of advertising in enhancing demand  $\theta$  is calibrated at 1.36, implying a long-run ratio of advertising expenditures to GDP

Table 4: Robustness

Model	Parameter Value ( $\gamma$ )	90% interval	Log Data Density
Hours in difference	0.584	[0.328 0.831]	1856.02
Measurement Error	0.561	[0.251 0.864]	1855.42
$\gamma = 1$	—	—	1852.95
Baseline	0.593	[0.354 0.822]	1855.18

Note: The measurement error in aggregate advertising is assumed to follow a disperse inverse-gamma distribution centered at 0.006.

of 0.029, as in the data. Following Ravn, Schmitt-Grohe and Uribe (2006), the government spending-GDP ratio,  $\bar{G}/Y$ , is set to 0.12, whereas the preference parameter  $\xi$  is chosen such that the steady-state value of hours worked,  $H$ , is equal to 0.25, thus implying that households devote 1/4 of their time to labor activities.

#### 4.4 Posterior distributions and estimation results

Jointly with the corresponding priors, Table 3 also reports posterior means and the 90% probability intervals for the estimated parameters. According to the reported results, all advertising-related parameters are estimated quite accurately. The data appear to be particularly informative regarding the competitiveness parameter,  $\gamma$ , whose posterior probability interval is approximately 50% smaller than its prior counterpart. Its posterior mean is 0.593, and the upper bound  $\gamma = 1$  lies well outside the 90% posterior probability interval, thereby indicating that the market-size channel of advertising is active in the U.S. economy. The data are also substantially informative with respect to the parameter that controls for the adjustment costs of advertising,  $\psi_z$ . Its posterior mean is equal to 1.227, suggesting a much faster response of advertising to shocks than what was assumed *a priori*.

As for the stochastic processes, we find that government shocks, labor supply shocks and economy-wide productivity shocks are quite persistent, with coefficients of autocorrelation ranging between 0.94 and 0.98. The advertising and investment-specific shocks show a milder persistence with autocorrelation coefficients equal to 0.84 and 0.75, respectively. This result is likely to be driven by the presence of adjustment costs, which generate endogenous persistence and therefore reduce the degree of exogenous autocorrelation needed to match the actual persistence in investment and advertising data. The estimates of the other structural parameters are consistent with the evidence available in the literature, with the exception of the habit persistent parameter ( $\zeta$ ), which is substantially lower than existing estimates (e.g., Smets and Wouters, 2007).

As Section 5 will clarify, the competitiveness parameter  $\gamma$  plays a central role in determining the effect of advertising on consumption and therefore we conduct a number of robustness checks on its estimates. First, we re-estimate the model using a series of hours worked in first differences instead of demeaned log levels. Second, we allow for measurement error in aggregate advertising. Finally, we estimate an alternative version of the model in which advertising is assumed to be purely competitive ( $\gamma = 1$ ).<sup>11</sup> The results are sum-

<sup>11</sup>The first robustness exercise controls for the assumption that the economy evolves along a balanced

Table 5: Second-order moments. Model, data and counterfactual analysis

	Model			Data	Counterfactual	
	Median	5%	95%		$\gamma = 0$	$\gamma = 1$
<b>Standard deviation (percentage)</b>						
GDP Growth ( $\Delta Y$ )	0.69	0.61	0.82	0.81	0.70	0.70
Investment Growth ( $\Delta I$ )	2.58	2.17	3.17	2.21	2.59	2.44
Demeaned Hours Worked ( $H$ )	2.03	1.37	3.22	2.46	2.07	2.00
Consumption Growth ( $\Delta C$ )	0.70	0.60	0.81	0.60	0.73	0.71
Advertising Growth ( $\Delta Z$ )	4.92	4.1	5.94	3.74	5.03	5.12
<b>Correlations</b>						
$\Delta C, \Delta Y$	0.48	0.29	0.64	0.64	0.47	0.50
$\Delta I, \Delta Y$	0.53	0.39	0.65	0.56	0.52	0.57
$H, \Delta Y$	0.09	-0.06	0.23	0.02	0.09	0.09
$\Delta Z, \Delta Y$	0.02	-0.12	0.22	0.22	0.07	0.05
$\Delta Z, \Delta C$	0.20	0.02	0.37	0.20	0.26	0.09
$\Delta Z, \Delta I$	-0.11	-0.29	0.1	0.19	-0.14	-0.02
$\Delta Z, H$	-0.05	-0.20	0.10	-0.04	-0.03	-0.03
<b>Autocorrelations</b>						
$\Delta Y$	0.18	0.00	0.38	0.37	0.18	0.19
$\Delta I$	0.62	0.46	0.74	0.49	0.75	0.66
$H$	0.93	0.86	0.97	0.96	0.97	0.92
$\Delta C$	0.19	0.03	0.35	0.20	0.17	0.20
$\Delta Z$	0.38	0.20	0.54	-0.31	0.36	0.41

Note: Model-based moments involved in the computation of the statistics reported in the table are estimated using the following procedure. First, we construct a random selection of 2,000 draws from the posterior distribution. Second, for each vector of parameters, we generate 100 artificial time series of the main variables of length equal to that of the data. Third, the resulting simulated series are used to compute second-order moments. Finally, summary statistics for the posterior distribution of moments are computed by pooling together all simulations. Moments for the counterfactual exercises are obtained by re-estimating the model under the restrictions  $\gamma = 0$  and  $\gamma = 1$ .

marized in Table 4. For all specifications, we report the estimated  $\gamma$ , the 90% posterior probability interval, and the marginal log data density. For the sake of convenience, the baseline estimation is reported in the last line. As the table illustrates, the estimate  $\hat{\gamma}$  is stable across specifications, and the 90% posterior probability intervals never include the

growth path in which hours worked are stationary. As noted by Chang, Doh and Schorfheide (2007), this theoretical restriction might be inconsistent with the data, potentially leading to biases in the estimates of the structural parameters and the shock processes. The second exercise controls for potential noise in our partial series of aggregate advertising. The last exercise is useful to compare the likelihood of the baseline model with the alternative specification in which the complementary property of advertising is switched off.

upper bound  $\gamma = 1$ . These findings, together with the fact that the restricted model with  $\gamma = 1$  yields the lowest marginal likelihood, confirm the results of the baseline estimation, thereby providing further evidence in support of the complementary effect of advertising.

Finally, we assess the ability of the model to fit the data by comparing second-order moments of the observable variables in the model with their counterparts in the data. According to Table 5, the model adequately replicates the main features of the data. All of data statistics fall into their corresponding 90% probability intervals, with the exception of the standard deviation and autocorrelation of advertising, which are both over-estimated. The fit of co-movements between advertising and the other aggregates is particularly accurate, showing that the model adequately replicates the pattern of cross-correlations measured in the data.

## 5 Quantitative results

In what follows, we use the estimated model to dwell on the long- and short-run implications of advertising by analyzing the steady-state equilibrium (Section 5.1) and aggregate dynamics (Section 5.2). The effect of advertising is disentangled by comparing the properties of the estimated model with those of an identical economy in which advertising is banned. We refer to this case as the *benchmark economy without advertising*.

### 5.1 Long-run effects

Panel A of Table 6 displays the estimated steady state of selected endogenous variables in percentage deviations from their benchmark counterparts.<sup>12</sup> According to the reported results, advertising stimulates an increase in consumption (6.79%), GDP (6.43%) and investment (4.28%); raises the markup (1.87%); and leads to a more consumption-based economy (the share of consumption in GDP increases by 0.40%). The strong increase in hour worked (10.9%) and the moderate decline in the wage rate (-2.47%) suggest that the underlying mechanism operates through a labor supply channel. In the presence of advertising, the labor supply schedule evaluated in the steady state is given by  $H = \Lambda (W/\xi^h)^{1/\phi} (C + (1 - \gamma)\theta F/(1 + \theta F))^{-\sigma/\phi}$ .<sup>13</sup> Hence, as long as  $\gamma \neq 1$ , an increase in advertising shifts the marginal rate of substitution between consumption and leisure toward the former. The household is then willing to work more to afford higher consumption levels, and therefore the labor supply schedule shifts to the right thus exerting downward pressure on real wages. In equilibrium, this mechanism triggers a twofold effect. On the one hand, firms expand their production, thus inducing a positive income effect that eventually increases consumption and investment. On the other hand, firms optimally substitute cap-

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<sup>12</sup>The steady state equilibrium is derived by setting each shock equal to its mean value and assuming that endogenous variables are constant over time. To account for parameter uncertainty, we construct a random section of 50,000 draws from the posterior distribution and compute the steady-state effects for each parameter vector. The summary statistics reported in Table 6 are obtained by pooling all simulations together.

<sup>13</sup>The labor supply is obtained using (B5) and (B7) evaluated at the steady state, and  $\Lambda = (1 - \beta\zeta/\tau)^{1/\phi} (1 - \zeta/\tau)^{-\sigma/\phi}$ .

Table 6: The Long-run effects of advertising

Model	Estimated ( $\gamma = 0.56$ )		Counterfactual ( $\gamma = 1$ )
	Median	90% interval	
<b>(A) Steady state effects (percentages)</b>			
Consumption	6.79	[0.42 17.3]	-1.40
Total Hours worked	10.9	[1.58 25.3]	0.90
Wage	-2.47	[-6.28 -0.25]	0
Production-related Hours	7.19	[0.56 18.5]	-1.40
GDP	6.43	[0.47 16.9]	-1.40
Investment	4.28	[0.05 12.2]	-1.40
Markup	1.87	[0.19 4.92]	0
Consumption Share	0.40	[0.04 1.02]	0
<b>(B) Welfare analysis</b>			
<b>(B.1) Steady state</b>			
$\tau_0$ (percentages)	-5.32	[-0.69 -16.2]	-
$\tau_z$ (percentages)	-1.22	[-0.16 3.59]	-
$H(Z)/H_e$	1.07	[1.01 1.17]	-
$C(Z)/C_e$	1.02	[0.96 1.12]	-
<b>(B.2) Transitional dynamics</b>			
$\tau_0$ (percentages)	-7.61	[-21.2 -3.55]	-
$\tau_z$ (percentages)	-2.80	[-5.51 0.211]	-

ital with labor, thus reducing the capital-output ratio. Eventually, the investment share in equilibrium decreases and this explains the observed increase in the share of consumption. Additionally, market-enhancing advertising is effective in shifting consumers' preferences and firms successfully exploit the wedge between perceived and true product differentiation by charging higher prices. As a result, the average markup also increases in equilibrium.

The mechanism linking advertising to consumption through the labor supply is known in the literature as the *work and spend cycle* and has been empirically supported by Baker and George (2010) for the U.S. and by Fraser and Paton (2003) for the U.K. Our results complement and extend this empirical evidence showing that, contrary to conventional wisdom, advertising does not necessarily stimulate consumption at the expense of savings. In fact, the estimated model argues that, at least in the long-run equilibrium, consumption and investment both increase with advertising, and contribute to fostering the economic activity.

Table 6 also compares the estimated effects of advertising with those implied by the economy with purely competitive advertising characterized in Proposition 1. The comparison highlights the key role of the complementarity property in determining both the sign and scale of the aggregate effects of advertising. In sharp contrast to the purely competitive

case, we find that (i) market-enhancing advertising is beneficial for GDP and its components and (ii) the effect of the *work and spend cycle* on hours worked is more than 10 times larger than the labor demand effect induced by purely competitive advertising. In the literature, the importance of complementary advertising was already postulated by Galbraith (1958), arguing that monopolistically competitive firms use advertising as a way to manipulate preferences with the aim of fostering high levels of consumption to increase profits. This process would affect agents' decisions regarding consumption and leisure, eventually increasing the supply of labor and the monopolistic power of firms and inducing a greater need for material consumption that makes the economy more consumption based. This view was subsequently criticized by Solow (1968) and Simon (1970). They argued that, because of its competitive nature, advertising affects the composition but not the size of aggregate consumption, therefore being irrelevant in the aggregate.<sup>14</sup> According to our estimated results, once both the complementary and competitive properties of advertising are accommodated in the model, the data indicate that the first property dominates the second and that the nature of aggregate spillover effects of advertising eventually closely resembles that conjectured by Galbraith.

What remains unclear from previous analysis is whether advertising is beneficial for the household. Advertising reduces welfare by inducing a higher markup that exacerbates the distortions of monopolistic competition, but it may also improve welfare by fostering consumption and hours worked, which potentially bring the economy closer to the competitive equilibrium. To assess the overall effect of advertising, we compare welfare in the estimated model to that of the benchmark economy without advertising. In the presence of persuasive advertising, the welfare analysis is complicated because advertising changes consumers' tastes. Therefore, it is not obvious which reference welfare criterion should be used given that there are at least two natural yardsticks: the pre-advertising and post-advertising preference relations.<sup>15</sup> We follow Benhabib and Bisin (2002) by relying on a welfare criterion that considers both preferences. Formally, let  $Z$  and  $U(C(Z), H(Z), Z)$  respectively denote the equilibrium level of aggregate advertising and the inter-temporal utility function associated with the allocation  $\{C(Z), H(Z)\} \in R_+^\infty$ . Then, a welfare criterion can be defined as follows.

**Definition 1.** *The household is better off in the presence of advertising if and only if the welfare increases with respect to post-advertising preferences,*

$$U(C(Z), H(Z), Z) \geq U(C(0), H(0), Z)$$

*and it also increases with respect to pre-advertising preferences,*

$$U(C(Z), H(Z), 0) \geq U(C(0), H(0), 0)$$

*with at least one inequality holding strictly.*

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<sup>14</sup>See Bagwell (2007) for references on the long-standing debate about aggregate advertising among classical economists.

<sup>15</sup>The issue is rather controversial in the literature. See Dixit and Norman (1978) and Benhabib and Bisin (2002) for a detailed discussion of this topic.

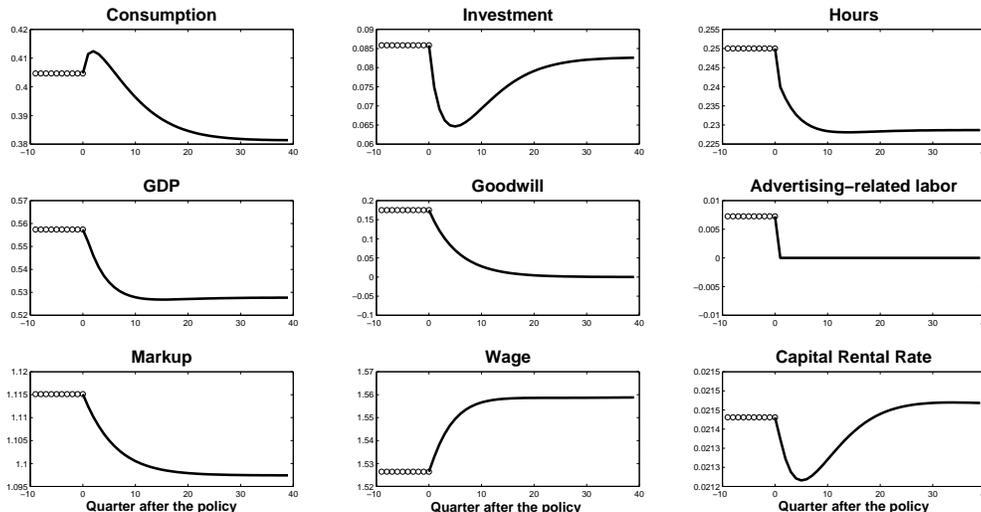


Figure 3: Transitional dynamics. The effects of a policy that completely bans advertising. Note: all of the parameters are set to their posterior mean values.

Definition 1 states that the consumer is better off with advertising ( $Z > 0$ ) if and only if he prefers the allocation  $\{C(Z), H(Z)\}$  to the allocation  $\{C(0), H(0)\}$ , regardless of which welfare yardstick is used. In particular, we compute the welfare gains of advertising as the percentage decrease in consumption that is required to make the household as well off as under a policy that totally bans advertising, i.e.,

$$U((1 - \tau_s)C(Z), H(Z), s) = U(C(0), H(0), s) \text{ with } s = \{0, z\}$$

where  $\tau_z$  and  $\tau_0$  denote the welfare gains associated with pre-advertising and post-advertising preferences, respectively. Hence, Definition 1 holds when both  $\tau_0 \geq 0$  and  $\tau_z \geq 0$  with at least one inequality holding strictly, whereas  $\tau_s < 0$  entails a welfare cost of advertising. The results are reported in panel B.1 and B.2 of Table 6. The first panel refers to the direct comparison of welfare between the pre and the post policy steady states. The second panel reports the results when computing the welfare costs along the whole transition from one steady state to the other. As far as the long-run equilibrium is concerned, we find that advertising is unambiguously detrimental to welfare because the household is worse off according to both ex ante and ex post preferences. For instance, with ex ante preferences a compensation in consumption of 5.32% is required to make households indifferent between the pre and post policy allocations. The origin of this result can be traced back to the overworking effect induced by advertising, which is apparent once comparing the steady state of the baseline model with that of a perfectly competitive equilibrium.<sup>16</sup> As reported in panel B.1, advertising pushes households to work 7% more hours while enjoying only 2% more consumption with respect to the efficient allocations, thus explaining the observed welfare losses. Panel B.2 shows that this welfare-detrimental effect is even stronger when

<sup>16</sup>Perfectly competitive allocations are determined from the benchmark model without advertising ( $\theta = 0$ ) by setting the average markup equal to 1.

the transitional dynamics is taken into account, in which case welfare costs raise up to 7.61%. Figure 3 illustrates that this amplification effect is due to the hump-shaped dynamics of consumption, which implies that for several quarters after the banning of advertising, households work less and consume more with respect to pre-policy steady state.

The hump-shaped consumption dynamics induced by the banning policy is an interesting result that deserves further explanations. To this end, the analysis of the transitional dynamics (Figure 3) highlights that this property is triggered by the carry-over effect of advertising, which induced a delayed response of consumption demand to the banning policy. The intuition is straightforward. After the prohibition of advertising, advertising-related labor jumps immediately to its new steady state ( $H_a = 0$ ) inducing a permanent downward shift in the labor demand, while the labor supply shifts upward because of the declining goodwill. As a result, total hours worked diminish, and this reduces the marginal productivity of capital so that the demand of capital also shifts to the left. Although this effect is partially dampen by the diminishing markup, the initial decline in the capital rental rate reveals that the shift in the demand of capital is sufficiently strong to induce a persistent excess of supply. Households react to this by substituting investment with consumption, and therefore the latter increases immediately after the banning policy. At the same time, the decline in the goodwill stock reduces the marginal utility of consumption, thereby driving downward pressure on the demand of consumption. However, since past advertising campaigns have a persistent influence on consumers' tastes (carry-over effect), it takes several quarters for consumption demand to completely adjust. As a result, the induced decline in consumption is delayed, which explains the hump-shaped pattern observed in Figure 3.

## 5.2 Short-run effects

### 5.2.1 Variance decomposition and advertising shocks

Panel A of Table 7 presents the results of the asymptotic variance decomposition for the estimated model (panel A.1) and for the counterfactual economy with purely competitive advertising (panel A.2). The reported contribution of the advertising-specific shock ( $\epsilon_t^z$ ) provides direct information on the the spillover effects from the advertising sector to the broader economy. The results show that exogenous shifts in advertising productivity contribute sensibly to fluctuations in consumption, investment and markup, respectively accounting for 16.9%, 12.1% and 90.4% of their volatility. On the contrary, their contribution to the volatility of total hours worked appears small (4.12%), and the contribution to the volatility in GDP is almost null (0.15%). The results reported in panel A.2 confirm the long-run analysis: the complementarity property and the associated market-enhancing effect are crucial for advertising to influence aggregate dynamics. When this property is switched off ( $\gamma = 1$ ), the contribution of advertising-specific shock to the volatility of all the considered aggregates falls virtually to zero.

To shed light on the mechanisms through which advertising affects the aggregate dynamics, Figure 4 reports the impulse response functions of some selected endogenous variables with respect to a 1% increase in the advertising-specific shock. A transitory boost in the productivity of advertising (i) raises consumption, hours worked, markup and advertising itself; (ii) induces a decline in investment and in the wage rate; and (iii) leaves the output

Table 7: Short-run Spillovers

	Variables				
	Consumption	Hours	Investment	GDP	Markup
<b>(A) Asymptotic variance decomposition</b>					
<b>(A.1) Baseline Estimation</b>					
Productivity ( $\epsilon_t^y$ )	32.4	6.20	19.3	49.2	4.11
Labor Supply ( $\epsilon_t^h$ )	21.0	64.0	13.1	33.7	2.32
Government ( $\epsilon_t^g$ )	13.6	14.0	5.31	9.10	1.40
Investment ( $\epsilon_t^k$ )	16.1	11.6	50.1	7.91	1.78
Advertising ( $\epsilon_t^z$ )	16.9	4.12	12.1	0.15	90.4
<b>(A.2) Purely competitive advertising (<math>\gamma = 1</math>)</b>					
Productivity ( $\epsilon_t^y$ )	36.2	5.16	20.9	51.0	0
Labor Supply ( $\epsilon_t^h$ )	24.6	71.0	13.9	34.4	0
Government ( $\epsilon_t^g$ )	16.9	12.4	5.89	7.28	0
Investment ( $\epsilon_t^k$ )	22.1	11.2	59.2	6.92	0
Advertising ( $\epsilon_t^z$ )	0.21	0.20	0.18	0.41	0
<b>(B) Historical contribution (change in volatility)</b>					
<b>(B.1) Baseline estimation</b>					
All shocks	23.9	16.0	-14.0	12.9	-
No advertising shocks	-11.6	5.1	12.6	14.1	-
<b>(B.2) Bounded marginal utility</b>					
All shocks	24.3	8.94	-14.4	6.60	-

Note: The statistics for the baseline estimation are computed by setting parameters to their posterior means values. The results for the counterfactual economy with purely competitive advertising are obtained by setting  $\gamma = 1$ , and holding all of the other parameters at their posterior mean values. Asymptotic variance decomposition refers to H-P filtered variables. Changes in volatility reported in the second row of panel B.1 (No advertising shocks) are computed from the estimated model by setting the advertising-specific shocks to zero. Results reported in Panel B.2 (Bounded marginal utility) are obtained by estimating an alternative version of the model where the consumption aggregate (2) is specified as in Molinari and Turino (2009).

virtually unaffected. Advertising increases because the shock reduces the marginal cost and this strengthens firms' incentives to expand their goodwill stocks. Furthermore, the presence of the adjustment costs implies that firms gradually adjust the goodwill stocks to the desired levels, and therefore the response of advertising to the shock is hump-shaped. The increase in advertising expenditures in turn raises the marginal utility of current consumption, at the impact of the shock and in all the sub-sequential quarters in which the goodwill stock increases. This is the key mechanism triggering the inter-temporal and intra-temporal effects of advertising. On the one hand, this mechanism makes the household less willing to smooth consumption through savings (urge to consume), thus driving a persistent decline

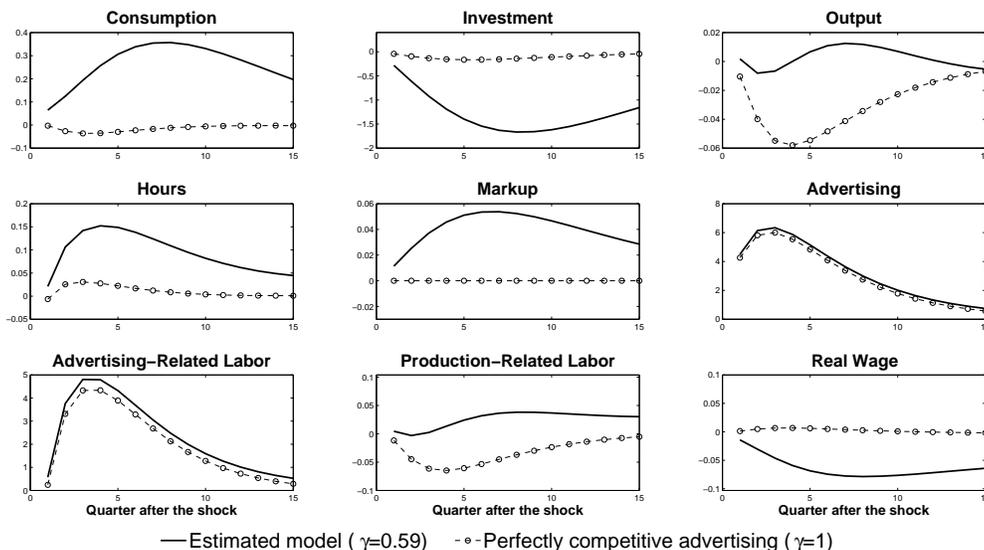


Figure 4: Impulse Response Functions of selected variables to a 1% increase in the productivity of advertising.

in investment accompanied by a positive and hump-shaped response of consumption that jointly determine the muted response of output. On the other hand, the same mechanism makes the household more willing to substitute leisure with consumption (work and spend mechanism), thus driving the increase in total hours worked and the decline in the real wage. As Figure 4 clarifies, however, the response of production-related labor is rather flat as result of the reallocation of labor to the advertising sector. This effect dampens the overall effect of advertising on total hours worked, thereby softening the quantitative implications of the *work and spend* channel on output fluctuations.

Figure 4 also reveals that the decline in output driven by the advertising-specific shock is accompanied by an expansion in the average markup. This finding hinges crucially on the presence of market-enhancing advertising, which is the necessary requirement for markups to be time-varying in the model.<sup>17</sup> In this case, the price-elasticity of demand for each good  $i$  can be expressed as

$$\eta_{y,p}(i) = \varepsilon (p_{i,t}/P_t)^{-\varepsilon} (\tilde{C}_t + I_t + F_t)/y_{i,t},$$

and shows that the demand of a product becomes more price elastic when the aggregate demand  $(\tilde{C}_t + I_t + F_t)$  increases. It follows from (10) that any shock enhancing aggregate demand also pushes firms to cut their markups, thereby triggering countercyclical movements in the average markup.<sup>18</sup> This mechanism is akin to the price-elasticity effect induced by the external deep habits formulation proposed by Ravn, Schmitt-Grohe and Uribe (2006). However, a crucial difference between the advertising model and the deep habits formula-

<sup>17</sup>This result follows from equations (12) and (13) which imply that either without advertising, or in the case of purely complementarity advertising, the price-elasticity of the demand is time-independent. Most likely, this is the reason why a substantial part of markup volatility (90%) in the estimated model is explained by the advertising-specific shock.

<sup>18</sup>Because of this mechanism, we find that markup and output are negatively correlated in the estimated model (-0.26). This finding is consistent with the available empirical evidence.

tion lies on the way customer bases are built in the two frameworks, and this has important implications for pricing policy. Under deep habits, current demand of each good depends positively on past sales. Hence, whenever a firm finds convenient to invest in customer base today, it will do it by cutting current markup. In our model, inter-temporal connections across sales are induced by advertising through the carry-over effect. In particular, a firm that wants to build a customer base for the future, has to increase advertising expenditures in the present. But an expansion in current advertising also induces a decline in the price elasticity of demand and this, in contrast with the deep habits formulation, pushes firms to increase current markups. Given that in our model advertising is pro-cyclical, this mechanism partially offsets the price-elasticity effect, thereby softening the implications of markup dynamics for output fluctuations.

For the sake of comparison, Figure 4 depicts the impulse response functions in the counterfactual economy with purely competitive advertising. As the figure illustrates, the market-enhancing effect is a necessary condition for advertising to trigger a positive and persistent response of consumption. Absent this effect ( $\gamma = 1$ ) the response of consumption to a boost in advertising productivity is negative and short-lived. In this case, advertising has no effect on the marginal utility and the model counterfactually predicts that consumption and advertising are uncorrelated over the business cycle. This finding suggests that the correlation of advertising with consumption in the data is most likely the source of identifying information for parameter  $\gamma$ . In this respect, note that the response of advertising is not affected by the value of  $\gamma$ , meaning that the parameter is not identified by the cyclical properties of advertising in the data (volatility and persistence). Some circumstantial evidence in support of this hypothesis is provided in the last two columns of Table 5, which report the second-order moments implied by re-estimating the model under the restrictions  $\gamma = 0$  and  $\gamma = 1$ . One of the most striking difference between the two restricted specifications occurs precisely in the case of the correlation between consumption and advertising growth. While in most of the reported statistics the difference is negligible, moving from competitive advertising ( $\gamma = 1$ ) to the purely complementary case ( $\gamma = 0$ ) has instead the effect of increasing the predicted correlation between consumption and advertising by 188%. Not surprisingly, this effect is mirrored in the correlation between advertising and investment growth, which also moves substantially with  $\gamma$  as result of the effects that aggregate advertising induces on the households' propensity to save. This finding further supports the hypothesis that the correlation between advertising and consumption in the data drives the identification of parameter  $\gamma$ .

### 5.2.2 The pro-cyclicity of advertising and the amplification mechanism

One crucial feature of the data presented in Section 2 is that advertising and consumption are positively correlated at the business cycle frequencies. The estimated model provides a natural tool to assess the sources of such co-movements. To this end, some useful insights can be elicited by analyzing the optimal advertising policy (11), which at the symmetric equilibrium can be written as

$$F_t = \Phi_t(1 - \mu_t^{-1})(\eta_{c,f}(t)C_t) \quad (14)$$

Table 8: Sources of advertising-consumption co-movements

	Estimation	Counterfactual					
		(A) Excluding shock:					(B)
		Advertising	Technology	Labor	Government	Investment	$\rho_{zy} = 0$
		$(\varepsilon_t^z)$	$(\varepsilon_t^y)$	$(\varepsilon_t^h)$	$(\varepsilon_t^g)$	$(\varepsilon_t^k)$	
$Corr(Z_t, C_t)$	0.61	0.39	0.29	0.54	0.65	0.68	0.34

Note: Predicted correlations are computed by setting the parameters to their posterior mean values and using the resulting smoothed estimates of the shocks to simulate model. The counterfactual correlation reported in Panel B refers to the case in which all shocks are active but parameter  $\rho_{zy}$  is set equal to 0.

where  $\eta_{c,f}(t) > 0$  stands for the elasticity of the demand of consumption goods with respect to the producers' goodwill, and  $\Phi_t = 1/[\nu_t - (1 - \delta_f) E_t \nu_{t+1} Q_{t,t+1}] > 0$ . Equation (14) shows that any shock increasing the aggregate demand of consumption  $C_t$  also pushes firms to expand their goodwill stocks. This is a direct source of co-movements between advertising and consumption, and captures that firms strategically use advertising to maintain market shares when the market demand increases. This effect can be either amplified or overturned by simultaneous changes in the term  $(1 - \mu_t^{-1})$ , depending on whether the shock expanding consumption ends up by decreasing or increasing the price-elasticity of demand. The reason is that the resulting movements in the optimal markup affects the marginal returns of an additional spending in advertising, and this may either strengthen or soften firms' incentives to expand goodwill stocks in response to a shock that busts consumption. Finally, the term  $\Phi_t$  captures the indirect sources of correlation between consumption and advertising, which are those co-movements driven by the effects of temporary technological changes and by movements in factors' prices.

To assess the relative importance of these difference sources of co-movements, we simulate the model by excluding each shock one at the time, and compute the implied correlation between consumption and advertising in each case. The results of this experiment are summarized in Table 8 where, for comparison, we also report the predictions for the baseline estimation (i.e. when all the shocks are active) as well as those implied by assuming idiosyncratic technological shocks (i.e. setting  $\rho_{zy} = 0$ ). As the table illustrates, the advertising-specific  $(\varepsilon_t^z)$  and the economy-wide technology  $(\varepsilon_t^y)$  shocks are the most important driving forces for the predicted correlation in the model, given that excluding these shocks reduces the statistics by 39% and 59% respectively, whereas for all the other shocks the effect is either negligible or much less important in magnitude. Interesting, the predicted correlation also declines substantially (- 44%) when the two technological components  $A_t^y$  and  $A_t^z$  are assumed to be idiosyncratic ( $\rho_{zy} = 0$ ). Figure 5 shows that this finding is driven by a resources reallocation effect that is akin to the standard mechanism of two-sectors models, according to which resources are re-allocated to the most productive sector after an idiosyncratic technological shock. Because of this mechanism, when  $\rho_{zy} = 0$ , advertising and consumption moves in the opposite direction after a boost in the economy-wide technology shock, and this explains why the predicted correlation between these two variables declines.

Another interesting result illustrated in Figure 5 is that the response of consumption

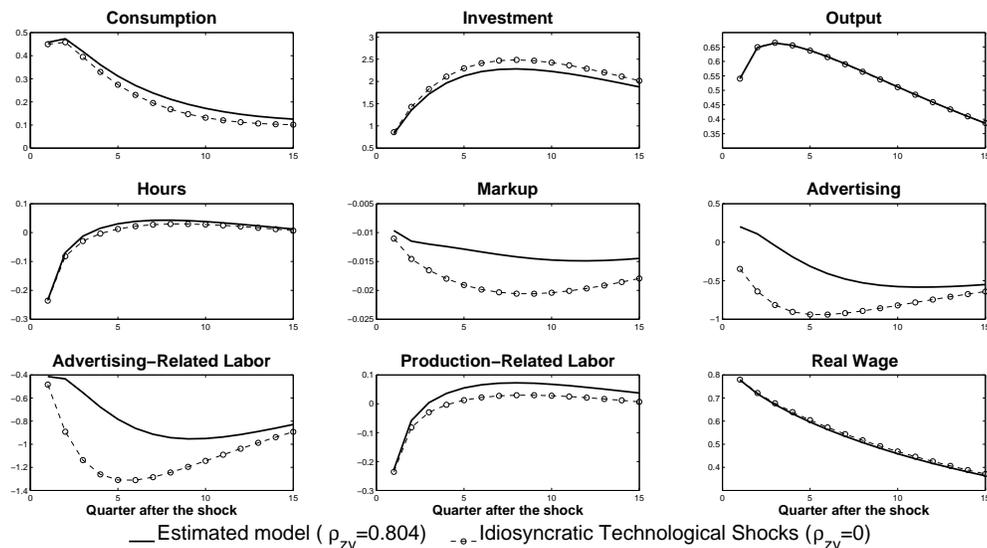


Figure 5: Impulse Response Functions of selected variables to a 1% increase in the economy-wide productivity shock.

in the estimated model is amplified with respect to the counterfactual economy with idiosyncratic productivities. This result captures the amplification mechanism of technology shocks induced by advertising. The intuition of this result is straightforward. When  $\rho_{zy} > 0$ , a shock increasing the economy-wide productivity ( $A_t^y$ ) also increases the productivity of the advertising sector ( $A_t^z$ ), thereby pushing firms to expand their goodwill stocks. The resulting increase in aggregate advertising raises the marginal utility, thus enhancing the response of consumption to the technology shock. Figure 5 also clarifies that the boosting effect of advertising on consumption occurs at the expense of savings, as apparent from the muted response of investment in the estimated model. By contrast, the response of hours worked is virtually identical and the countercyclical response of markup is dampen. These results suggest that the inter-temporal substitution is the only channel through which advertising amplifies the effects of a technology shock.

### 5.2.3 Quantifying the spillover effects

There is not a unique way to quantify the overall spillover effects of advertising. One possibility is to compute in-sample estimates of the historical role played by advertising in shaping the U.S. aggregate dynamics. To this end, Figure 6 presents predicted consumption and investment (continuous lines) along with their counterparts implied by the benchmark model without advertising (dashed lines), whereas Panel B.1 of Table 7 (first row) reports the in-sample variance of selected endogenous variables in percentage deviations from their benchmark counterparts.<sup>19</sup> As the figure illustrates, in the estimated model fluctuations in

<sup>19</sup>The reported results are obtained by setting the parameters to their posterior mean values and using the resulting smoothed estimates of the shocks to simulate both the baseline and benchmark models. To facilitate comparisons, the series reported in Figure 6 are normalized to equal 1 in the initial quarter. Predictions for the benchmark model without advertising are obtained from the baseline model by setting

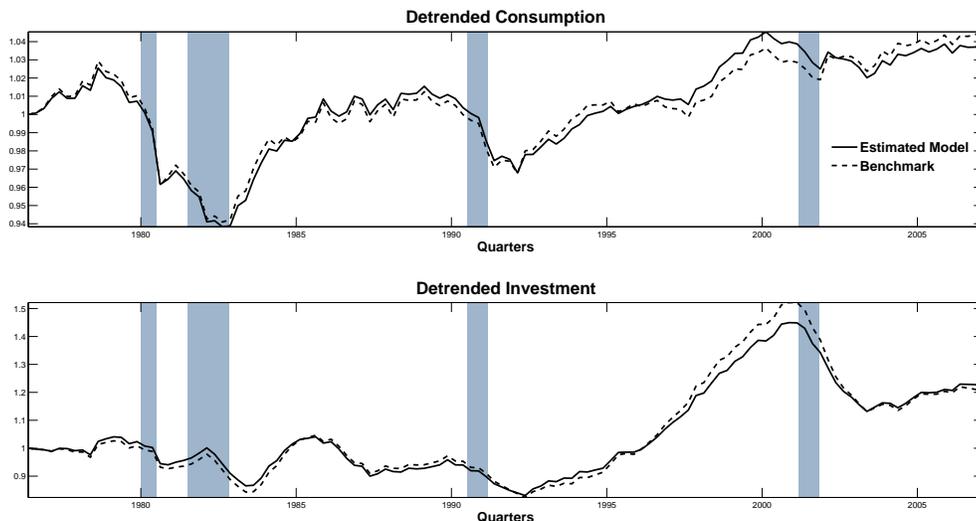


Figure 6: Predicted consumption and investment. Shaded areas refer to NBER recessions.

consumption are generally amplified. Because of that, we find that consumption volatility increases by 23.9% with advertising. Investment shows an opposite pattern, and in fact, in-sample volatility decreases by 14%, suggesting that the inter-temporal substitution effect induced by advertising is a crucial driving force for the spillover effects on consumption. We also find that the presence of advertising increases the volatility of both total hours worked (16%) and GDP (14.1%), but the results are quantitatively less important in comparison with consumption.

The presence of the advertising-specific shock turns out to be crucial in determining the amplification effect on consumption. This finding is illustrated in panel B.1 of Table 7 (second row), which shows that in the absence of the advertising-specific shock the estimated model predicts that consumption volatility declines by 11.6%. The reason is that the enhancing effect of advertising on the propensity to consume in the economy works as a dampening mechanism of any shock that crowds out consumption. This property is apparent in Figure 7, which shows that in the cases of government and investment shocks, consumption is less responsive in the model with advertising, and this effect drives the observed decline in consumption volatility. This finding also indicates that the measured change in volatility does not completely capture the boosting effect of advertising on consumption. In this respect, we can use equation (4) to compute the elasticity of aggregate demand of consumption to advertising as alternative measure of the spillover effects of advertising. By using the in-sample predictions of the model, we find that the statistics takes an average value of 0.10, thus implying that the demand of aggregate consumption increases by 10% when advertising is doubled.<sup>20</sup>

parameter  $\theta = 0$ , whereas all of the other parameters are kept fixed to their posterior mean values.

<sup>20</sup>Specifically, by keeping  $\tilde{C}_t$  fixed, we use equation (4) at the symmetric equilibrium to compute the elasticity of consumption demand to advertising. The resulting equation is then used with the model predictions on goodwill stock ( $F_t$ ), advertising ( $Z_t$ ) and consumption ( $C_t$ ) to quantify the predicted elasticity over the sample-period. In this computation, all of the parameters are set to their posterior mean values.

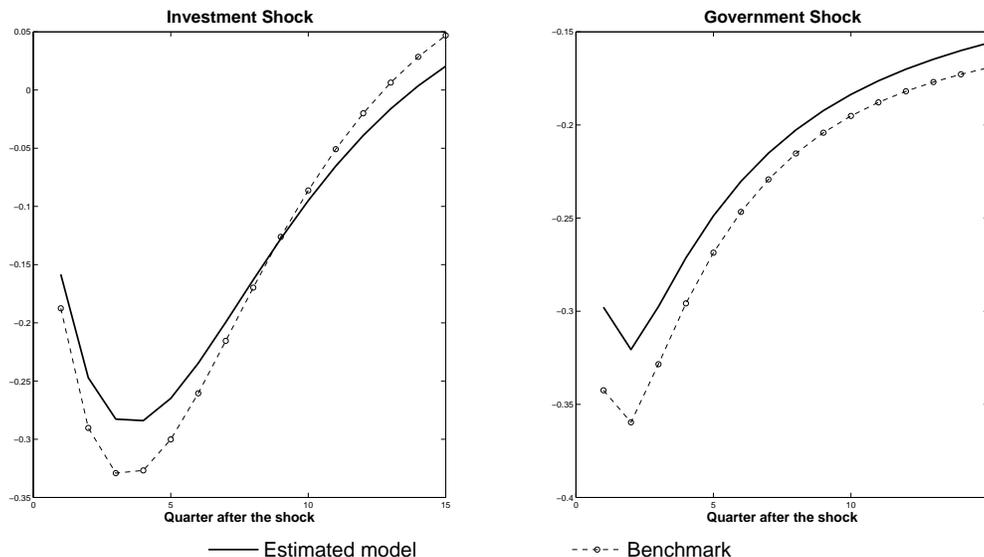


Figure 7: The effect of government and investment specific shock on consumption. Note: all of the parameters are set to their posterior mean values.

We conclude this section by testing the robustness of our findings. As emphasized in Section 4.2, the specification of function  $B(\cdot)$  implies that without advertising, the consumption aggregate (2) takes the standard Dixit-Stiglitz form and therefore, in the benchmark model, markup stays constant over the business cycle. Given that markup is instead countercyclical in the estimated model, it follows that the amplification effects documented in Table 7 may be artificially over-estimated by the assumptions made on  $B(\cdot)$ . To control for this issue, we have estimated an alternative version of our model where we adopt the consumption aggregate with bounded marginal utility proposed by Molinari and Turino (2009). This formulation implies that the markup is time-varying independently on the presence of advertising, and therefore allows us to control for markup fluctuations in disentangling the spillovers effects of advertising. Results are summarized in panel B.2 of Table 7, where for selected endogenous variables we report the predicted changes in volatility. In comparison with the baseline results (panel B.1), we find that controlling for markup fluctuations is inconsequential for the predicted effects on consumption and investment, thereby showing that the inter-temporal substitution mechanism induced by advertising is robust with respect to the assumptions made on function  $B(\cdot)$ . By contrast, the amplification effect of advertising on both hours worked and GDP appears to be significantly over-estimated in the baseline specification. To some extent, these findings can be interpreted as suggesting that the *work and spend* mechanism may play a secondary role in driving the short-run effects of advertising.

## 6 Conclusions

This paper assesses the role of marketing practices in stimulating consumption and economic activity. By estimating a variant of the neoclassical growth model that accounts for

firms' spending on advertising with U.S. data, we show that advertising has a relevant long-run effect on consumption and work activities. The mechanism operates through a *work and spend* channel: in the presence of advertising, people work more to afford larger purchases of goods, and the perceived need for higher consumption results from the advertising signals to which consumers are exposed. Because of this effect, advertising enhances the production of output, eventually increasing consumption and investment. We also show that in the short run, the effect of the work and spend channel on output and hours worked is offset by the counter-work of an intertemporal substitution mechanism. Households substitute savings with current consumption, and the resulting reduction in aggregate investment offsets the increase in consumption, maintaining production close to the that observed in a counterfactual economy in which advertising is banned. In fact, we find that advertising affects the business cycle primarily through the dynamics of consumption and investment.

Finally, it should be noted that the flexible-price one-sector model developed in this paper uses the worst-case scenario for advertising to influence economic activity. In fact, in a model with nominal rigidities, the *work and spend* channel can be enhanced because the lower wage variability might determine a larger increase in consumers' supply of labor in response to advertising shocks. Moreover, in a one-sector model, any increase in markup due to advertising makes investment more expensive, therefore reducing the real return on capital and decreasing savings. As a result, the effect of advertising on investment is negative in the short run, further smoothing the overall effect of advertising on economic activity.

## References

- Aaker, David A., and Robert Jacobson.** 1994. "The Financial Information Content of Perceived Quality." *Journal of Marketing Research*, 31(2): pp. 191–201.
- An, Sungbae, and Frank Schorfheide.** 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2-4): 113–172.
- Ashley, R, C W J Granger, and R Schmalensee.** 1980. "Advertising and Aggregate Consumption: An Analysis of Causality." *Econometrica*, 48(5): 1149–67.
- Bagwell, Kyle.** 2007. "The Economic Analysis of Advertising." , ed. Mark Armstrong and Robert Porter Vol. 3 of *Handbook of Industrial Organization*, Chapter 28, 1701–1844. Elsevier.
- Baker, Matthew J., and Lisa M. George.** 2010. "The Role of Television in Household Debt: Evidence from the 1950's." *The B.E. Journal of Economic Analysis & Policy*, 10(1).
- Benhabib, Jess, and Alberto Bisin.** 2002. "Advertising, Mass Consumption and Capitalism." New York University Mimeo.
- Bisin, Alberto, and Jess Benhabib.** 2010. "Social Construction of Preferences: Advertising." , ed. Jess Benhabib, Alberto Bisin and Matt Jackson Vol. 1A of *Handbook of Social Economics*, 201–220. Elsevier.

- Blank, David M.** 1962. "Cyclical Behavior of National Advertising." *The Journal of Business*, 35(1): 14–27.
- Chang, Yongsung, Taeyoung Doh, and Frank Schorfheide.** 2007. "Non-stationary Hours in a DSGE Model." *Journal of Money, Credit and Banking*, 39(6): 1357–1373.
- Chowdhury, Abdur R.** 1994. "Advertising expenditures and the macroeconomy: Some new evidence." *International Journal of Advertising*, 13(1): 1–14.
- Clarke, Darral G.** 1976. "Econometric Measurement of the Duration of Advertising Effect on Sales." *Journal of Marketing Research*, 13(4): 345–357.
- Deighton, John, Caroline M. Henderson, and Scott A. Neslin.** 1994. "The Effects of Advertising on Brand Switching and Repeat Purchasing." *Journal of Marketing Research*, 31(1): pp. 28–43.
- Dixit, Avinash, and Victor Norman.** 1978. "Advertising and Welfare." *Bell Journal of Economics*, 9(1): 1–17.
- Dorfman, Robert, and Peter O. Steiner.** 1954. "Optimal Advertising and Optimal Quality." *American Economic Review*, 44(5): 826–836.
- Fraser, Stuart, and David Paton.** 2003. "Does advertising increase labour supply? Time series evidence from the UK." *Applied Economics*, 35(11): 1357–1368.
- Galbraith, John K.** 1958. "The Affluent Society." Houghton Mifflin Company.
- Grossmann, Volker.** 2008. "Advertising, in-house R&D, and growth." *Oxford Economic Papers*, 60(1): 168–191.
- Hall, Robert E.** 2008. "General Equilibrium with Customer Relationships: A Dynamic Analysis of Rent-Seeking."
- Iacoviello, Matteo, and Stefano Neri.** 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics*, 2(2): 125–64.
- Jacobson, Robert, and Franco M. Nicosia.** 1981. "Advertising and Public Policy: The Macroeconomic Effects of Advertising." *Journal of Marketing Research*, 18(1): 29–38.
- Jung, Chulho, and Barry J. Seldon.** 1995. "The Macroeconomic Relationship between Advertising and Consumption." *Southern Economic Journal*, 61(3): 577–587.
- Kamakura, Wagner A., and Gary J. Russell.** 1993. "Measuring brand value with scanner data." *International Journal of Research in Marketing*, 10(1): 9–22.
- Karray, Salma, and Guiomar Martn-Herrn.** 2009. "A dynamic model for advertising and pricing competition between national and store brands." *European Journal of Operational Research*, 193(2): 451–467.

- Leone, Robert P.** 1995. "Generalizing What is Known about Temporal Aggregation and Advertising Carryover." *Marketing Science*, 14(3): pp. G141–G150.
- McClure, Samuel M., Tomlin Damon, Li Jian, P. Read Montague, Kim S. Cypert, and Latane M. Montague.** 2004. "Neural Correlates of Behavioral Preference for Culturally Familiar Drinks." *Neuron*, 44(2): pp. 28–52.
- Molinari, Benedetto, and Francesco Turino.** 2009. "Advertising and Business Cycle Fluctuations." Instituto Valenciano de Investigaciones Económicas, S.A. (Ivie) Working Papers. Serie AD 2009-09.
- Nerlove, Marc, and Kenneth J. Arrow.** 1962. "Optimal Advertising Policy under Dynamic Conditions." *Economica*, 29(114): 129–142.
- Ravn, Morten, Stephanie Schmitt-Grohe, and Martin Uribe.** 2006. "Deep Habits." *Review of Economic Studies*, 73(1): 195–218.
- Roberts, Mark J., and Larry Samuelson.** 1988. "An Empirical Analysis of Dynamic, Nonprice Competition in an Oligopolistic Industry." *RAND Journal of Economics*, 19(2): 200–220.
- Seldom, Barry J., and Chulho Jung.** 1995. "The length of the effect of aggregate advertising on aggregate consumption." *Economics Letters*, 48(2): 207–211.
- Simon, Julian L.** 1970. "Issues in the Economic of Advertising." University of Illinois Press.
- Smets, Frank, and Rafael Wouters.** 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.
- Solow, Robert M.** 1968. "The truth further refined: A comment on Marris." *Public Interest*, 11: 47–52.
- Stern, B. L., and A. J. Resnik.** 1991. "Information Content in Television Advertising: A Replication and Extension." *Journal of Advertising Research*, 31(2).
- Taylor, Lester D, and Daniel Weiserbs.** 1972. "Advertising and the Aggregate Consumption Function." *American Economic Review*, 62(4): 642–55.
- Vakratsas, Demetrios, and Tim Ambler.** 1999. "How Advertising Works: What Do We Really Know?" *The Journal of Marketing*, 63(1): 26–43.
- Vakratsas, Demetrios, Fred M. Feinberg, Frank M. Bass, and Gurumurthy Kalyanaram.** 2004. "The Shape of Advertising Response Functions Revisited: A Model of Dynamic Probabilistic Thresholds." *Marketing Science*, 23(1): 109–119.
- Yoo, Boonghee, Naveen Donthu, and Sungho Lee.** 2000. "An examination of selected marketing mix elements and brand equity." *Journal of the Academy of Marketing Science*, 28: 195–211.

# Appendix

## A Data

Data on Aggregate Advertising			
Source	Media	Frequency	Sample
Robert J. Coen	All Media	Annual	1948-2007
AD\$Summary journal	All Media	Quarterly	1976-2006
Newspaper Association of America	Newspapers	Quarterly	1976-2006

### Business Cycle Indicators

Macroeconomic Variable	Code	Freq.	Sample
Real GDP	GDPC96	Quarterly	1976.I-2006.II
Real Export	EXPGSC96	Quarterly	1976.I-2006.II
Real Import	IMPGSC96	Quarterly	1976.I-2006.II
Real Personal Consumption Exp.	PCECC96	Quarterly	1976.I-2006.II
Real PCE: Durable Goods	PCDGCC96	Quarterly	1976.I-2006.II
Real PCE: Nondurable Goods	PCNDGC96	Quarterly	1976.I-2006.II
Real Private Fixed Investment	FPIC96	Quarterly	1976.I-2006.II
GDP Implicit Price Deflator	GDPDEF	Quarterly	1976.I-2006.II
Civ. Non-Instit. Population	CNP160V	Quarterly	1976.I-2006.II
Civ. Employment-Pop. Ratio	EMRATIO	Quarterly	1976.I-2006.II
Average weekly hours of employees	CES05007*	Quarterly	1976.I-2006.II

*All Media* refers to Magazines, Sunday Magazines, Outdoor, Network Television, Spot Television, Cable Television, Syndicated Television, Network Radio, Spot radio, Newspapers.

*Data sources:* (Coen) <https://spreadsheets.google.com/pub?key=p9LENaiKJeoyBX4eR1FZEEw>; (AD\$Summary) printed issues available at the library of the Johnson Graduate School of Management, Cornell University; (NAA) [www.naa.org](http://www.naa.org); Business Cycle Indicators. (FRED II of the Federal Reserve Bank of St. Louis) [research.stlouisfed.org/fred2](http://research.stlouisfed.org/fred2); \*(CES of U.S. Bureau of Labor Statistics) [www.bls.gov/ces](http://www.bls.gov/ces)

## B The model equations

This appendix summarizes the system of equations describing the decentralized equilibrium of the economy. The equilibrium price of goods is normalized to 1 in each period and, accordingly, the price index is:  $P_t = 1 \forall t$ . Then, using the optimal demand for investment  $i_{i,t} = (p_{i,t}/P_t)^{-\varepsilon} I_t$  and consumption goods (equation (3)), the budget constrain for the representative household can be written as

$$C_t + I_t + T_t = W_t H_t + R_t u_t \bar{K}_t + \Pi_t - a(u_t) \bar{K}_t$$

where the capital utilization cost is given by  $a(u_t) = R_{ss}^k (\omega u_t^2/2 + (1 - \omega)u_t + \omega/2 - 1)$  with  $R_{ss}^k$  denoting the steady state rental rate of capital. The representative household chooses

sequence of consumption ( $C_t$ ), hours worked ( $H_t$ ) and capital utilization ( $u_t$ ) by maximizing the inter-temporal utility function (2) under the budget constraint and the capital law of motion

$$\bar{K}_{t+1} = (1 - \delta_k) \bar{K}_t + \xi_t^k (1 - \phi_{k,t}) I_t \quad (\text{B1})$$

where the capital adjustment cost is specified as  $\phi_{k,t} = \frac{\psi_k}{2} (I_t/I_{t-1} - \gamma_I)^2$  with  $\gamma_I$  denoting the growth rate of investment in the balanced growth path equilibrium. The first order conditions for an interior solution of the utility maximization problem are:

$$\lambda_t = \Gamma_t^{\sigma-1} (\tilde{C}_t - \zeta \tilde{C}_{t-1})^{-\sigma} - \Gamma_{t+1}^{\sigma-1} \beta \zeta E_t \left\{ (\tilde{C}_{t+1} - \zeta \tilde{C}_t)^{-\sigma} \right\} \quad (\text{B2})$$

$$\lambda_t^k = \beta E_t \left\{ (1 - \delta_k) \lambda_{t+1}^k + (R_{t+1}^k u_{t+1} - a(u_{t+1})) \lambda_{t+1} \right\} \quad (\text{B3})$$

$$\lambda_t = \lambda_t^k \left[ \xi_t^k \left( 1 - \phi_{k,t} - \frac{\partial \phi_{k,t}}{\partial I_t} I_t \right) \right] - \beta E_t \left\{ \lambda_{t+1}^k \xi_{t+1}^k \frac{\partial \phi_{k,t+1}}{\partial I_t} I_{t+1} \right\} \quad (\text{B4})$$

$$\lambda_t W_t = \xi_t^h H_t^\phi \quad (\text{B5})$$

$$R_t^k = a'(u_t) \quad (\text{B6})$$

where  $\lambda_t$  and  $\lambda_t^k$  respectively denote the Lagrange multipliers associated with the budget constraint and the capital law of motion. Using equation (12), the consumption aggregate (2) can then be written as

$$\tilde{C}_t = C_t - B(F_t, F_t, \Gamma_t) \equiv C_t - (1 - \gamma) \Gamma_t \left[ \frac{\theta F_t / \Gamma_t}{1 + \theta F_t / \Gamma_t} \right] \quad (\text{B7})$$

At the symmetric equilibrium, total output ( $Y_t$ ) and aggregate advertising ( $Z_t$ ) are, respectively, given by

$$Y_t = A_t^y K_{i,t}^{1-\alpha} (\Gamma_t H p_t)^\alpha \quad (\text{B8})$$

$$Z_t = A_t^z \Gamma_t H a_t^{\alpha z} \quad (\text{B9})$$

where  $K_t$  denotes effective capital that evolves according to

$$K_t = u_t \bar{K}_t \quad (\text{B10})$$

Aggregate advertising accumulates into the goodwill stock ( $F_t$ ) according to the law of motion

$$F_t = (1 - \delta_f) F_{t-1} + (1 - \phi_{z,t}) Z_t \quad (\text{B11})$$

where the advertising adjustment cost is specified as  $\phi_{z,t} = \frac{\psi_z}{2} (Z_t/Z_{t-1} - \gamma_z)^2$  with  $\gamma_z$  denoting the growth rate of advertising in the balanced growth path. The first order conditions for profit-maximizing firms are:

$$\mu_t = \frac{\varepsilon (1 - B(\cdot)/Y_t)}{\varepsilon (1 - B(\cdot)/Y_t) - 1} \quad (\text{B12})$$

$$\varphi_t = \mu_t^{-1} \quad (\text{B13})$$

$$R_t^k = \varphi_t (1 - \alpha) Y_t / K_t \quad (\text{B14})$$

$$W_t = \varphi_t \alpha Y_t / H_{p,t} \quad (\text{B15})$$

$$\nu_t^z = \alpha_z^{-1} (W_t / Z_t) H_{a,t} \quad (\text{B16})$$

$$\nu_t^z = \nu_t \left[ (1 - \phi_{z,t}) - \frac{\partial \phi_{z,t}}{\partial Z_t} Z_t \right] - \beta E_t \left\{ \nu_{t+1} \frac{\partial \phi_{z,t+1}}{\partial Z_t} Z_{t+1} \right\} \quad (\text{B17})$$

$$(1 - \varphi_t) B_{f_i}(F_{t+1}, F_{t+1}, \Gamma_{t+1}) + (1 - \delta_f) E_t \{ \nu_{t+1} Q_{t,t+1} \} = \nu_t \quad (\text{B18})$$

where  $Q_{t,t+1} = \beta(\lambda_{t+1}/\lambda_t)^{-\sigma}$  is the stochastic discount factor, while  $B_{f_i}(F_t, F_t, \Gamma_t)$  denotes the partial derivative of  $B(\cdot)$  with respect to its first argument and evaluated at the aggregate goodwill stock,  $F_t$ . Market clearing requires that the following conditions hold

$$H_t = H_{a,t} + H_{p,t} \quad (\text{B19})$$

$$Y_t = C_t + I_t + G_t + a(u_t) \bar{K}_t \quad (\text{B20})$$

while fiscal revenues are determined on a balanced basis, according to which  $T_t = G_t$ . The government spending evolves according to  $G_t = \Gamma_t (G_{t-1} / \Gamma_{t-1})^{\rho_g} \bar{G}^{(1-\rho_g)} \exp\{\epsilon_t^g\}$  where  $\rho_g \in [0, 1)$ ,  $\epsilon_t^g$  is an i.i.d. innovation with a mean of 0 and standard deviation  $\sigma_g$ , and  $\bar{G}$  is detrended government spending ( $G_t / \Gamma_t$ ) evaluated in the steady state. Furthermore, in equilibrium dividends paid to households read as  $\Pi_t = (1 - \varphi_t) Y_t - \alpha_z \nu_t^z Z_t$  while GDP for this economy is defined as

$$GDP_t = C_t + I_t + G_t \quad (\text{B21})$$

Letting  $X_t = \{F_t, \mu_t, Z_t, H_t, H_{a,t}, H_{p,t}, C_t, \bar{K}_{t+1}, I_t, Y_t, R_t^k, W_t, \tilde{C}_t, u_t, K_t, \nu_t^z, \nu_t, \lambda_t, \lambda_t^k, GDP_t, \varphi_t\}$  denote the vector of endogenous variables and  $\vartheta_t = \{A_t^y, A_t^z, \xi_t^k, \xi_t^h, G_t\}$  be the vector of exogenous state variables, a symmetric equilibrium can then be formally defined as a pair of initial conditions  $(\bar{K}_0, F_0) \in R_+^2$  and a sequence  $\{X_t\}_{t=0}^\infty$  that solves equations (B1)-(B21) given the stochastic process  $\{\vartheta_t\}_{t=0}^\infty$ .

## C Proof of Proposition 1

Let us denote by  $C_t^d$  the aggregate demand of consumption that results from equation (4) by keeping fixed the level of  $\tilde{C}_t$ . Assume in addition that advertising is purely competitive so that changes in aggregate goodwill  $F_t$  do not affect the demand of aggregate consumption, i.e.  $\partial C_t^d / \partial F_t = 0 \forall F_t \geq 0$  and  $t$ . Given that at the symmetric equilibrium  $C_t^d = c_{i,t} \forall i \in [0, 1]$ , the previous condition implies that  $\partial c_{i,t} / \partial F_t = 0 \forall i$ .<sup>21</sup> This last property can equivalently be stated as  $B(F_t, F_t, \Gamma_t) = B(0, 0, \Gamma_t) = 0 \forall F_t \geq 0$  and  $\forall t$ , and implies that the equilibrium level of the average markup, being equal to  $\mu_t = \varepsilon / (\varepsilon - 1) \forall t$ , is also independent of advertising (see equation B12).

Let us refer hereafter to  $X_{CA}$  and  $X_{NA}$  as the steady states of  $X$  in the model economy with purely competitive advertising and in the benchmark model without advertising, respectively. Assume furthermore that  $\tau = 1$  so that the system of equations (B1)-(B20) describes the equilibrium in the de-trended model. Thus, the finding that  $\mu_t = \varepsilon / (\varepsilon - 1)$

<sup>21</sup>In other words, purely competitive advertising leaves the demand function of each firm unaffected in the symmetric equilibrium.

$\forall t$  shows that  $\mu_{CA} = \mu_{NA}$ . In addition, equations (B2) and (B3) prove the steady state capital rental rate is the same in the two economies, i.e.  $R_{CA}^k = R_{NA}^k$ . Therefore, given equations (B1), (B8), (B14), (B15) and (B20), it is also true that  $(Y/K)_{CA} = (Y/K)_{NA}$ ,  $(H_p/K)_{CA} = (H_p/K)_{NA}$ ,  $(C/K)_{CA} = (C/K)_{NA}$  and  $W_{CA} = W_{NA}$ . Now, notice that at the steady state the Lagrange multiplier  $\lambda$  can be written

$$\lambda_j = \Lambda C_j^{-\sigma} \text{ with } j = \{CA, NA\} \text{ and } \Lambda = (1 - \zeta)^{-\sigma}(1 - \beta\zeta) \quad (15)$$

By using this result with the intra-temporal condition (B5), we find

$$\begin{aligned} H_{CA}^{\sigma+\phi} &= \Lambda((C/K)_{CA})^{-\sigma}((H/K)_{CA})^\sigma W_{CA}/\xi^h \\ &> \Lambda((C/K)_{CA})^{-\sigma}((H_p/K)_{CA})^\sigma W_{CA}/\xi^h \\ &= \Lambda((C/K)_{NA})^{-\sigma}((H_p/K)_{NA})^\sigma W_{NA}/\xi^h \\ &= H_{NA}^{\sigma+\phi} \end{aligned}$$

where the first inequality follows from the labor market clearing condition (B19). Therefore,  $H_{CA} > H_{NA}$ . This last result implies that  $\lambda_{CA} > \lambda_{NA}$  because otherwise, given that  $W_{CA} = W_{NA}$ , the intra-temporal condition (B5) would be not satisfied. As a result, because of equation (15),  $C_{CA} < C_{NA}$ . But then  $(C/K)_{CA} = (C/K)_{NA}$  implies  $K_{CA} < K_{NA}$ , which leads to  $Y_{CA} < Y_{NA}$  and  $I_{CA} < I_{NA}$  because of  $(Y/K)_{CA} = (Y/K)_{NA}$  and equation (B1). This proves the statement.