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“PROGRESSIVE TAXATION AND CORPORATE LIQUIDATION: ANALYSIS AND POLICY IMPLICATIONS”

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Abstract

This paper contributes to the debate on alternative corporate tax schemes, employing a rigorous real option methodology which has never been used to study both liquidation policy and taxation. Different tax systems are considered, according to whether the tax regime is progressive or flat and losses are deductible or not. The critical liquidation threshold is derived as a function of interest expenses, the firm’s driving parameters and the tax rates and taxation brackets. It is shown that only the adoption of a flat tax plan does not interfere with the firm’s liquidation policy, while any progressive tax schedule can slow down or speed up the closure policy.

Keywords: Corporate debt, default risk, progressive tax, real options.

JEL Code: G3, G32, G33, G12, H2, H32

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1 Introduction

In the debate among tax policy specialists there is no consensus on the appropriate rate structure reforms that should be undertaken. In an open economy, characterized by interdependencies between national and regional systems, integration becomes more difficult both by progressivity in income taxation and by the differential tax treatment of different types of assets. The choice of a tax scheme involves careful evaluation about its economic impact in terms of efficiency and the incentive consequences of marginal tax rates. In several important contributions (see Sørensen 1998) it has been argued that the Nordic tax system - based on a dual income tax regime, in which all sources of asset income are taxed at a common rate - avoids the distortions and improves efficiency. The recent experience of numerous countries adopting a flat tax has renewed the debate among the advocates and detractors of proportional taxation (see Grecu 2004). A flat income tax applies the same tax rate to all tax-payers and to each income component. Without personal exemptions the tax is proportional, with personal exemptions it becomes progressive, so that taxation increases with income although marginal taxation remains constant. In 1994 Estonia introduced a flat tax on personal and corporate income at a rate of 26%, now at 23%. Latvia and Lithuania followed suit. In 2001 Russia unified its marginal rates of personal income taxation at a single rate of 13%. Various reforms applying flat taxation have been followed in 2003 in Serbia (now at a 14% rate), Ukraine (25%), Slovakia (19%), then in 2005 in Georgia (20%) and Romania (16%) and finally in Bulgaria, which is now the European country with the lowest tax rate (10%). Reforms are considered also in Belarus, the Kyrgyz Republic, Poland and in countries in Latin America, where the debate
has played itself out again and again. Ivanova, Keen and Klemm (2005) provide a most interesting discussion of recent flat tax proposals. The gist of it is that the Russian reform, by producing an increase in revenue of about 26% and being accompanied by a strong enforcement policy, has worked quite well. Given the importance of these influential reforms, it is worthy to provide a clear assessment of the effects of alternative taxation rules. The purpose of this paper is to provide a contribution in this debate focusing on a specific issue, that is, to understand how different tax rules affect corporate liquidation policy and corporate behavior.

Modigliani and Miller (1963) demonstrated that in a perfect frictionless world firms do not have optimal tax-driven capital structure, so that the value of a firm with debt is equal to the value of an identical firm without debt. Yet firms do have to trade off the tax benefits of debt with the cost of financial distress (Jensen and Meckling 1976, Myers 1977). As it has been stressed by De Angelo and Masulis (1980) and more recently reviewed by Graham (1996, 2003) and Eckbo (2004), the tax benefit of debt can be crowded out by non-debt tax shields; moreover, firms do not always benefit from incremental tax deductions when taxable income is negative. As a consequence, a careful treatment of the loss limitation rules is called for together with an appropriate evaluation of tax and non-tax shields to measure the effective marginal tax rates and to link tax status with corporate debt policy.

We assume a simple tax structure that includes personal as well as corporate taxes and where interest expenses are deductible. Our model incorporates both tax and non-tax based explanations of corporate disinvestment policy, such as bankruptcy costs and agency costs. Moreover, three different tax rules are considered to deal with negative income: (i) no deductibility for losses; (ii) full loss offset provisions; (iii) asymmetric tax rates for positive or negative tax income. We assume that the firm’s tax position cannot switch among the different taxation schemes over time. The different tax schemes are compared to study how they affect the firm’s liquidation trigger. Liquidation usually occurs in the context of taxable losses. These typically introduce an asymmetry which is addressed in this paper. If the tax base is smaller than the tax exemption, the tax-payer gets a payment from the fiscal authorities. Such "negative tax" is maximum under rule (ii), where the tax rate applied for the tax reimbursement is the same for positive and negative income. However, the tax system typically provides less than full loss offset, not giving tax refund to investors with negative current taxable income. Under (iii) the tax system treats positive and negative income asymmetrically, because two different rates are applied for the tax reimbursement, with a smaller one for negative income. Finally, the simplest such asymmetry is rule (i), where the "negative tax" is zero, since the rate for the tax reimbursement is zero with negative income. Our paper clearly indicates the consequences of a negative tax base in the light of a proportional and progressive tax schedule and shows the importance of tax rules with respect to the choice of the financial structure, corporate policies and their implications for policy-makers’ analysis. While most papers have investigated the impact of tax rules on investment decisions (see Bettendorf 1996 for international interactions.
and Auerbach 2002 for a thorough survey), our paper deals with corporate liquidation policies. Only a few papers have considered the effect of tax advantage on a firm’s liquidation policy (see, for example, Leland 1994, Goldstein, Ju, Leland 2001, Fischer, Heinkel and Zechner 1989). Goldstein, Ju and Leland (2001) develop a dynamic model of capital structure, where capital gains are taxed on accrual, and study the effect of tax advantages to debt when future debt levels can be increased. However, the taxation scheme they consider is proportional, while in our paper we model a richer taxation structure, both evaluating proportional vs progressive taxation and incorporating alternative tax rules for the tax reimbursement against losses. Their paper has some similarity to Graham (2000), which simulates interest deduction benefits and uses them to estimate the tax-reducing value of debt, although this latter is numerically based and does not permit equity holders to choose bankruptcy. In a related paper, Fischer, Heinkel and Zechner (1989) formulate a dynamic model that predicts that the tax benefits to debt are mostly negligible and are an increasing function of firm value volatility, although both results are contradicted by Graham’s findings.

Our aim is to provide a closed-form solution for debt and equity values and for the liquidation threshold, which is accomplished throughout the real options approach we adopt. Real options represent the formal modeling technique which is more appropriate to serve the purposes of decision making in a dynamic context under uncertainty. Traditional static approaches are not suitable in the study of liquidation policies, because of the high uncertainty and costs of irreversible decisions; moreover, they underestimate the upside potentials of the operational strategies. Real options methods allow to incorporate all these elements of corporate policy. Within such a methodology we integrate corporate taxation as well as equity holders’ and debt holders’ individual taxation, extending the use of this model for decision making and for policy-makers. Indeed, real option literature has been enriched by taxation only recently to display taxational distortions of investment decisions (Agliardi 2001, Niemann and Sureth 2004). Our paper instead applies real option methodology to a liquidation problem. Most papers in the existing literature assume taxation to be proportional. Our result compares a flat tax schedule with a progressive tax system, where tax progression means that the average tax rate increases with the tax base. We prove that the introduction of tax exemptions or tax credits, which make the marginal tax rate and the average tax rate unequal, do play a role in determining disinvestment and may yield inefficiency in the liquidation policy, in some circumstances. We investigate the interplay between the size of the marginal tax rate and the fiscal treatment of gains and losses as determinants of corporate closure policy and show that only under a "perfect" flat tax system the amount of the marginal tax rate does not affect the liquidation trigger value. Our analysis builds on the literature analyzing how capital structure is affected by the costs due to conflicts of interest between equity holders and debt holders. Conflicts between debt holders and equity holders may arise because of the equity holders’ incentive to invest in risky but poor projects, resulting in a decrease in the value of the debt. In case of liquidation of the firm, the firm’s
management receives nothing and thus has an incentive to postpone disinvestment even if liquidation is preferred by investors. A larger debt level improves the liquidation decision because it makes default more likely. Our work explores the extent to which the taxation rules themselves lead to inefficient results, both in terms of unsecured creditors’ claims and welfare losses to society. Therefore it has public policy implications in that it suggests that the adoption of a proper tax code might facilitate socially efficient economic outcomes.

This paper is organized as follows. Section 2 describes the model set-up. Sections 3, 4 and 5 investigate the effect of the different corporate tax schemes i), ii), iii) in details. Section 6 contains a comparative analysis of the corporate financial policy under different tax regimes and concludes.

2 The basic model

Let us consider the behavior of a representative firm whose net operating profit per period is denoted by $V$. Assume, as usual in real option analysis, that $V$ follows the stochastic process $dV = \mu dt + \sigma dW$, where $W$ is a standard Brownian motion, $\mu$ is a drift term and $\sigma$ measures the volatility. Let $r$ denote the risk-free interest rate, with $r > \mu$ since we consider the case of falling operating profits which might trigger closure.

We confine ourselves to the case of a firm managed by the shareholders, so that possible conflicting effects due to governance mechanisms are ignored in order to focus on tax effects. The firm is financed by issuing equity and debt. Debt service consists of a perpetual coupon payment $C$. If the asset value decreases to a level $V$ such that the firm defaults on the coupon payment, then liquidation occurs. Closure is irreversible and the debt holders receive the liquidation value of the firm’s stock of capital, net of bankruptcy costs. Let $K$ denote the liquidation value of the firm’s stock of capital. Bankruptcy costs are assumed to be proportional to the liquidation value and let $\alpha$, $0 \leq \alpha \leq 1$, measure the proportion. If bankruptcy occurs, a fraction $0 \leq \alpha \leq 1$ of value is lost because of bankruptcy costs, leaving debt holders with a reduced value. Debt holders are senior claimants, while equity holders are left with nothing as a residual claim. In what follows we denote by $D(V)$ and $E(V)$ the values of the claims of the debt holders and equity holders. We aim at determining the values of these claims and the closure threshold $\bar{V}$. In the argument below we suppose that $\bar{V} \geq 0$: then, it is implicitly assumed that, whenever our argument fails, such a $\bar{V}$ does not exist, that is, the firm never closes down. Let $d(V)$ and $e(V)$ be the payout policies to debt holders and equity holders, respectively. If $\tau_b$ denotes the personal tax rate for bond investment income, then the payout policy to bondholders is $d(V) = C(1 - \tau_b)$, if $V > \bar{V}$ and $d(V) = (1 - \alpha)rK$, if $V \leq \bar{V}$. Firms may deduct interest payments from corporate taxable income. If bankruptcy is declared corporate payments are made in the following order: interest

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1Lewis (1990) argues that taxes are irrelevant to debt maturity choice. Guedes and Opler (1996) find little support for tax-based theories of debt maturity choice.
payments, corporate tax payments and any residual payments to stockholders.
It is assumed that the net capital gain deduction is ruled out in the tax code,
while some form of net capital losses deduction is retained. In the following
sections we are going to study how different tax environment affect the liquidation policy of the firm. As a consequence, the cash flow to equity holders, \( e(V) \),
will take on different expressions, depending on the corporate taxation scheme.
Three corporate taxation rules will be examined, that is: i) no deductibility for losses, ii) full loss offset provisions, iii) asymmetric tax rate for positive or
negative taxable income. We will assume that the firm cannot switch among
the different taxation regimes over time, but is permanently in one or another.
In this section we develop a basic case which serves as a framework for the subsequent specifications. For the moment, we abstract from the real expression of
the net (after-tax) cash flow to equity holders and let \( e(V) \) be a generic function
satisfying \( e(V) = 0 \) for \( V \leq \bar{V} \) and some broad technical assumptions which are
detailed below. In the following sections \( e(V) \) will be specialized according to
the three different tax rules mentioned above. Proposition 1 proves an analytical formula for the values of the debt holders’ and equity holders’ claims and
provides a general rule to determine the liquidation threshold \( \bar{V} \). Since the firm
is managed by equity holders who set the debt policy and the closure policy, in
the subsequent analysis \( \bar{V} \) and \( E(V) \) assume different expression depending on
the tax environment in which the firm operates, while the expression for \( D(V) \)
remains unchanged and therefore will not be mentioned any more.

**Proposition 1** Suppose that there exists \( \bar{V} \geq 0 \) such that the firm closes down
as soon as \( V \) reaches the liquidation threshold \( \bar{V} \). Let \( e(V) \) denote the net (after
taxes) cash flow to equity holders and let \( e(V) \) be a generic function satisfying \( e(V) = 0 \) for \( V \leq \bar{V} \) and some broad technical assumptions which are
detailed below. In the following sections \( e(V) \) will be specialized according to
the three different tax rules mentioned above. Proposition 1 proves an analytical formula for the values of the debt holders’ and equity holders’ claims and
provides a general rule to determine the liquidation threshold \( \bar{V} \). Since the firm
is managed by equity holders who set the debt policy and the closure policy, in
the subsequent analysis \( \bar{V} \) and \( E(V) \) assume different expression depending on
the tax environment in which the firm operates, while the expression for \( D(V) \)
remains unchanged and therefore will not be mentioned any more.

\[
\int_{\bar{V}}^{+\infty} e(Y)Y^{-1-\lambda^+} dY = 0
\]  
\[\text{(1)}\]

where \( \lambda^\pm \) are the roots of the characteristic equation \( \sigma^2 \lambda^2 + (2\mu - \sigma^2)\lambda - 2r = 0 \),
\( \lambda^- < 0 < \lambda^+ \). Moreover, the values of the debt holders’ and equity holders’
claims are given by:

\[
D(V) = \frac{C(1-\tau_b)}{r} \left( 1 - \frac{V}{\bar{V}} \right)^{\lambda^-} + (1-\alpha)K \left( \frac{V}{\bar{V}} \right)^{\lambda^+} \quad \text{for } V > \bar{V}
\]
\[
= (1-\alpha)K \quad \text{for } V \leq \bar{V}
\]

\[
E(V) = \frac{2}{\sigma^2(\lambda^+ - \lambda^-)} \left\{ V^{\lambda^-} \int_{\bar{V}}^{V} e(Y)Y^{-1-\lambda^-} dY + V^{\lambda^+} \int_{V}^{+\infty} e(Y)Y^{-1-\lambda^+} dY \right\}
\]
\[
= 0 \quad \text{for } V \leq \bar{V}
\]  
\[\text{(2)}\]
Proof. Following the Dixit and Pindyck (1994) framework we see that the equity holders’ claim $E(V)$ satisfies the following differential equation for $V > \overline{V}$, where $\overline{V}$ is the liquidation threshold that will be determined below:

$$\frac{1}{2} \sigma^2 V^2 \frac{d^2 E}{dV^2} + \mu V \frac{dE}{dV} + e(V) = rE$$

The general solution is

$$H_1 V^{\lambda^-} + H_2 V^{\lambda^+} \frac{2}{\sigma^2 (\lambda^- - \lambda^+)} \left\{ V^{\lambda^-} \int_{Y}^{V} e(Y) Y^{-1-\lambda^-} dY + V^{\lambda^+} \int_{V}^{\infty} e(Y) Y^{-1-\lambda^+} dY \right\}$$

where $\lambda^-$ and $\lambda^+$ are the solutions to $\sigma^2 \lambda^2 + (2 \mu - \sigma^2) \lambda - 2r = 0$, $\lambda^- < 0 < \lambda^+$. In view of the no-bubble condition for $V \to \infty$ we have $H_2 = 0$. Finally we apply the smooth-pasting conditions $E(\overline{V}) = 0$ and $\frac{d}{dV} E(\overline{V}) = 0$. Combining the two equations we get $H_1 = 0$ and we find that the optimal closure threshold satisfies (1). Then the equity holders’ claim can be written as (2), where $\overline{V}$ satisfies the condition (1). The value $D(V)$ of a claim on the payout policy to the bondholders $d(V)$ satisfies the following differential equation for $V > \overline{V}$:

$$\frac{1}{2} \sigma^2 V^2 \frac{d^2 D}{dV^2} + \mu V \frac{dD}{dV} + C(1-\tau_b) = rD$$

whose general solution is $\frac{C(1-\tau_b)}{\sigma^2} + \overline{H}_1 V^{\lambda^-}$, if we take the no-bubble condition $\lim_{V \to \infty} D(V) = \frac{C(1-\tau_b)}{\sigma^2}$ into account. Then $\overline{H}_1$ is determined employing the boundary condition $D(\overline{V}) = (1-\alpha)K$. If $V < \overline{V}$, then $D(V) = (1-\alpha)K + \overline{H}_2 V^{\lambda^+}$, where $\overline{H}_2 = 0$ in view of the pasting condition $D(\overline{V}+) = D_0(V-)$.

Observe that $(\overline{V})^{\lambda^-}$ has the interpretation of a measure of the probability of bankruptcy. Thus, the value of the debt is the sum of the face value of the firm multiplied by the probability that the firm is solvent and the expected net liquidation value at bankruptcy. The expression of the value of equity will become clearer in the following sections, where the basic model is specialized to different tax schemes as regards the taxation on corporate gain or losses.

3 The case of no deductibility for losses

Let us consider the following taxation regime, including both personal and corporate taxes. Let $\tau_e$ be the personal tax rate for stock investment income and $\tau_c$, the marginal corporate tax. We allow corporate taxation to be progressive and model tax progression by means of an exogenously given tax exemption threshold $x \geq 0$. If $x > 0$ at least two tax brackets are included in the taxation scheme and thus the corporate taxation is progressive, because, due to tax exemption, the average tax rate increases with the tax base even if the marginal tax rate is constant. Alternatively $\phi_c = \tau_c x$ may be interpreted as a fixed tax credit, thereby inducing an element of indirect progressivity. In this section we assume that the tax system gives no tax refund to those investors with negative current taxable income, that is, we confine ourselves to the case of no deductibility for
losses. For brevity’s sake, this tax rule is referred to as "ND". Therefore, the after-tax cash flow to equity holders is:

\[ c(V) = (V - C - \tau_c \max(V - C - x, 0))(1 - \tau_c) \quad \text{if} \quad V > \overline{V} \quad ("ND") \]
\[ = 0 \quad \text{if} \quad V \leq \overline{V} \]

where \( \overline{V} \) denotes the firm’s liquidation trigger value. The expression of the payout policy above implements a mark-to-market taxation rule. That is, in each period, capital gains are taxed as accrued even though they are not realized, like most papers where income is taxed continuously (see Fischer, Heinkel and Zechner 1989, Goldstein, Ju and Leland 2001, Auerbach 2002). The following proposition holds:

**Proposition 2** Under the corporate taxation scheme "ND" the closure threshold is the solution \( \overline{V} \leq C \) to the following equation:

\[ \frac{\overline{V}^{-\lambda^+}}{1 - \lambda^+} + \frac{C \overline{V}^{-\lambda^+}}{\lambda^+(1 - \lambda^+)} = \frac{\tau_c(C + x)^{1 - \lambda^+}}{\lambda^+(1 - \lambda^+)} \]

Moreover, the value of the equity holders’ claim \( E(V) \) for \( V > \overline{V} \) is:

\[
\begin{cases}
\frac{V}{\lim_{\tau \rightarrow \tau_c}} - \frac{C}{r} + \frac{2}{\sigma^2(\lambda^+ - \lambda^-)} \left\{ \frac{\tau_c(C + x)}{\lambda^+(1 - \lambda^+)} \left( \frac{V}{C + x} \right)^{\lambda^+} - \left( \frac{V}{1 - \lambda^-} + \frac{C}{\lambda^-} \right) \left( \frac{V}{C + x} \right)^{\lambda^-} \right\}(1 - \tau_c) & \text{if} \quad \overline{V} < V < C + x \\
\frac{V(1 - \tau_c)}{\lim_{\tau \rightarrow \tau_c}} - \frac{C(1 - \tau_c)}{r} + \frac{2}{\sigma^2(\lambda^+ - \lambda^-)} \left( \frac{\tau_c(C + x)}{\lambda^+(1 - \lambda^+)} \left( \frac{V}{C + x} \right)^{\lambda^+} - \left( \frac{V}{1 - \lambda^-} + \frac{C}{\lambda^-} \right) \left( \frac{V}{C + x} \right)^{\lambda^-} \right) & \text{if} \quad V \geq C + x.
\end{cases}
\]

**Proof.** In view of Proposition 1 the closure threshold is obtained solving the following equation for \( \overline{V} \):

\[
\int_{\overline{V}}^{+\infty} (Y - C - \tau_c \max(Y - C - x, 0)) Y^{-1 - \lambda^+} dY = 0
\]

Working out this expression when \( c(V) \) is given by ("ND") we obtain that \( \overline{V} \) is a solution to \( \frac{\overline{V}^{-\lambda^+}}{1 - \lambda^+} + \frac{C \overline{V}^{-\lambda^+}}{\lambda^+(1 - \lambda^+)} - \frac{(C + x)^{1 - \lambda^+}}{\lambda^+(1 - \lambda^+)} = 0 \), with \( \overline{V} \leq C + x \). Let \( f(\overline{V}) \) denote the left-hand side of this equation. Since \( f(0^+) = +\infty \), \lim_{\overline{V} \rightarrow \infty} f(\overline{V}) > 0 \) and \( f \) has a negative minimum in \( C + x \), then the equation \( f(\overline{V}) = 0 \) admits two solutions. Only the solution less than \( C \) is compatible with the condition \( \overline{V} \leq C + x \). Finally, in view of Proposition 1, the following expression for the equity holders’ claim obtains:

\[
2 \frac{(1 - \tau_c)}{\sigma^2(\lambda^+ - \lambda^-)} (V^{-\lambda^+}) \int_{\overline{V}}^{+\infty} (Y - C - \tau_c \max(Y - C - x, 0)) Y^{-1 - \lambda^+} dY
\]

which yields our final expression for \( E(V) \). \( \blacksquare \)
Let us now investigate the effect of this form of taxation on the liquidation decision. First denote the closure threshold in absence of taxation by $V^0_0$. Then $V^0_0 = \frac{(\lambda-1)C}{\lambda}$ that can also be written in the form $-\frac{\lambda-C}{1-\lambda} \frac{\mu}{r}$, as in Agliardi and Agliardi (2007). With the same notation as in the proof of Proposition 2 we note that $f(V) < 0$ for any $V \in [\overrightarrow{\xi},C]$ and $f(V) > 0$ for any $V \in [0,\overrightarrow{\xi}]$. Therefore from $f(V^0_0) > 0$ we get $V^0_0 < \overrightarrow{\xi}$. Furthermore the following estimates hold: $\frac{\partial^2 V}{\partial x^2} > 0$, $\frac{\partial V}{\partial x} < 0$ and $\frac{\partial V}{\partial C} > 0$. We can paraphrase these results in the following Corollary ND:

**Corollary ND.** (i) The introduction of a corporate taxation "ND" speeds up closure; (ii) the effect is emphasized by a higher marginal rate, while the presence of tax exemption brackets has a dampening effect; (iii) debt speeds up closure.

**Policy implications.** Under "ND" policy-makers aiming at reducing the occurrence of bankruptcy should either reduce the marginal tax rate or introduce larger tax exemption brackets.

### 4 Full loss offset provision

In this section we study almost opposite tax rules providing tax benefits for firms with losses and no limit deductions for interest. If these deduction induce negative taxable income, the tax system provides tax refunds. We consider a tax system providing symmetric treatments of positive and negative earnings after interest. In the previous section the simplest case of tax asymmetry - no deductibility for losses - has been analyzed, while next section will consider an intermediate case.

With the same notation as in the previous section we write down the after-tax payoff to equity holders as follows:

$$e(V) = ((V-C)(1-\tau_e)+x\tau_e)(1-\tau_e) \quad if \ V > \overrightarrow{\xi}$$

$$= 0 \quad if \ V \leq \overrightarrow{\xi}$$

("FD")

where $\overrightarrow{\xi}$ denotes the firm’s liquidation trigger value. Such a tax system will be shortly denoted by "FD" (full deductibility). Note that the presence of $x > 0$ induces a progressive taxation, while for $x = 0$ the tax scheme is genuinely proportional. Thus, this case is the most helpful to highlight the different impact of progressive versus flat taxation. Applying Proposition 1 to this case we obtain:
Proposition 3 Under the tax system "FD" and whenever \( C \geq \frac{x_c}{1-\tau_c} \) there exists a nonnegative liquidation threshold \( V = \frac{1-\lambda}{\lambda^+} \left[ \frac{\tau_c x_c}{1-\tau_c} - C \right] \). Moreover, the value of the shareholders’ claim for \( V > 0 \) is given by:

\[
E(V) = \left( \frac{1-\tau_c}{r-\mu} \right) (V - \bar{V})^{\lambda^-} + \left( \frac{x\tau_c - C(1-\tau_c)}{r} \right) (1-\tau_e)(1-(\bar{V})^{\lambda^-})
\]

Observe that, if \( x_c > C(\frac{1}{1-\tau_c} - 1) \), then \( \bar{V} < 0 \), that is, in presence of sufficiently large tax exemptions the firm never closes. Anyway \( \bar{V} < V_0^0 \), where \( V_0^0 \) is the closure threshold in absence of taxation. In more intuitive terms, the introduction of taxation "FD" slows down closure. Importantly, \( V_0^0 \) equals also the liquidation threshold when no tax exemption bracket is provided for by the tax code, that is, when \( x = 0 \). Finally, we observe that \( \bar{V} \) increases with \( C \). Therefore, the following Corollary FD holds:

**Corollary FD.** (i) Under regime FD the closure policy is unaffected by a proportional tax plan, that is, the closure threshold \( V_0^0 \) remains the same, however high may be the marginal tax rate; (ii) on the contrary, the adoption of a progressive tax does interfere with the firm’s liquidation policy, resulting in inefficiently later closure; (iii) debt speeds up closure.

**Policy implications.** A neutral policy toward corporate liquidation requires a flat tax combined with full deductibility. A progressive tax plan always generates inefficiency in terms of welfare, since it induces a delayed closure and reduces the value of the debt holders’ claim in liquidation.

5 Asymmetric treatment of positive or negative taxable income

In Section 4 we assumed implicitly that interest payments be deductible at the corporate rate regardless of their magnitude. On the other hand, in Section 3, we limited deductions for interest to the extent that these deductions would yield positive taxable income. In this section we take an intermediate view, by supposing that the tax code treats positive and negative taxable income values asymmetrically, that is the corporate tax rate is \( \tau_c \) for positive tax base and \( \tau_c^* \) for negative ones, with \( 0 < \tau_c^* < \tau_c \). To study the effect of tax progression we introduce two tax brackets for positive tax base and denote the threshold between them by \( x \). Symmetrically, we consider a level \( x^* \leq x \), such that a refund (at the rate \( \tau_c^* \)) applies whenever income falls below it. Generally, \( x^* = 0 \). In the case \( x^* < 0 \) we confine ourselves to \( C + x^* > 0 \), that is, a limit is set on interest deduction that is used to offset poor corporate income. Adopting the same notation as in the previous sections, the after-tax payoff to equity holders is written as follows
\(e(V) = (V - C) - \tau_c \max(V - C - x, 0) + \tau_c^* \max(V - C - x, 0)\) if \(V > \overline{V}\)
\(= 0\) if \(V \leq \overline{V}\)

where \(\overline{V}\) denotes the firm’s liquidation trigger value. To avoid complication, the personal tax rate for stock investment income, \(\tau_c\), has been omitted in this Section. This tax system will be shortly denoted by "AS" (Asymmetry).

In view of Proposition 1 the corporate liquidation threshold is determined as a solution \(\overline{V} \leq C + x^*\) to the following equation:

\[
\frac{V^{1-\lambda^+}(1-\tau_c^*)}{1-\lambda^+} + \frac{V^{\lambda^+}(C(1-\tau_c^*)-\tau_c^*x^*)}{\lambda^+} = \frac{x^*(C+x^*)^{1-\lambda^+} - \tau_c^*(C+x^*)^{1-\lambda^+}}{\lambda^+}
\]

(3)

In the sequel we distinguish two cases \(x^* \leq C(1/\tau_c^* - 1)\). Let \(g(\overline{V})\) denote the left-hand side of this equation. If \(x^* \geq C(1/\tau_c^* - 1)\), then \(g\) increases from \(-\infty\) to 0 and thus (3) admits a unique solution \(\overline{V} > 0\) whenever \(\tau_c^* < \tau_c\left(\frac{\overline{V}+C}{\overline{V}^*}\right)^{\lambda^+-1}\). Moreover, \(\frac{\partial g}{\partial \overline{V}} < 0\), \(\frac{\partial g}{\partial C} > 0\), \(\frac{\partial g}{\partial x^*} > 0\). If \(x^* < C(1/\tau_c^* - 1)\), then \(g\) has a negative minimum at \(C - \frac{\tau_c^*}{1-\tau_c^*}\) and (3) always admits a solution \(\overline{V}\). The sign of the derivatives of \(\overline{V}\) is reversed with respect to the previous case.

**Proposition 4** Under the corporate taxation scheme "AS" the closure threshold is a solution \(\overline{V} \leq C + x^*\) to the equation (3). As far as the impact of taxation on the liquidation policy is concerned, this case splits into two subcases depending on \(x^* \leq C(1/\tau_c^* - 1)\). If \(x^* \geq C(1/\tau_c^* - 1)\) then tax benefits speed up closure, while a high tax rate slows down closure, that is, the behavior is like in "FD". If \(x^* < C(1/\tau_c^* - 1)\) then tax benefits slow down closure, while a high tax rate speeds up closure, as in "ND".

Note that the tax system "ND" can be obtained as a limit case of "AS" letting \(\tau_c^* \to 0\). On the other hand, if \(\tau_c^* \to \tau_c\) and \(x^* = x\), then "AS" boils down to "FD".

Finally, the equity value \(E(V)\) is:

\[
\frac{V(1-\tau_c^*)}{\tau - \mu} - \frac{C(1-\tau_c^*)-\tau_c^*x^*}{\tau} + \frac{V^2(1-\tau_c^*)}{\tau(1-\tau_c^*)} \left( \frac{V}{C+x^*} \right)^\lambda^+ + \tau_c^*(C+x^*).
\]

\[
\left( \frac{V}{\lambda^+} \right)^\lambda^+ - \frac{1}{\lambda^+} \left( \frac{V}{\lambda^+} \right)^\lambda^+ - \left( \frac{V(1-\tau_c^*)}{\tau} + \frac{\tau_c^*(C+x^*)}{\lambda^+} \right)^\lambda^+ \right]
\]

if \(\overline{V} < V \leq C + x^*\)

\[
\frac{V}{\tau - \mu} - \frac{C}{\sigma^2(\lambda^+ - \lambda)} \left( \frac{\tau_c^*(C+x^*)}{\lambda^+} \right)^\lambda^+ + \frac{\tau_c^*(C+x^*)}{\lambda^+} - \tau_c^*(C+x^*)
\]

\[
\left( \frac{V}{\lambda^+} \right)^\lambda^+ - \left( \frac{V(1-\tau_c^*)}{\tau} + \frac{\tau_c^*(C+x^*)}{\lambda^+} \right)^\lambda^+ \right]
\]

if \(C + x^* < V \leq C + x\)

\[
\frac{V(1-\tau_c^*)}{\tau - \mu} - \frac{C(1-\tau_c^*)-\tau_c^*x^*}{\tau} + \frac{2}{\sigma^2(\lambda^+ - \lambda)} \left( \frac{\tau_c^*(C+x^*)}{\lambda^+} \right)^\lambda^+ + \frac{\tau_c^*(C+x^*)}{\lambda^+} - \tau_c^*(C+x^*)
\]

\[
\left( \frac{V}{\lambda^+} \right)^\lambda^+ + \left( \frac{\tau_c^*(C+x^*)}{\lambda^+} - \frac{V(1-\tau_c^*)}{\tau} \right)^\lambda^+ \right]
\]

if \(C + x < V\).
Policy implications. Proposition 4 shows how an appropriate combination of marginal tax rates and tax refunds should be used by policy-makers either to promote or to restrain the event of liquidation.

6 Concluding remarks

In order to compare the effects of the different taxation regimes a numerical simulation is performed. In the basic model we take $r = 4\%$, $\mu = 2\%$, $\sigma = 30\%$, $\alpha = 5\%$, $\tau_c = 33\%$, $\tau_c^* = 10\%$, $\tau_b = 12.5\%$. Table 1 reports numerical results.

<table>
<thead>
<tr>
<th>Simulated values</th>
<th>No tax</th>
<th>ND</th>
<th>FD</th>
<th>AS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V/V$</td>
<td>0.41</td>
<td>0.50</td>
<td>0.33</td>
<td>0.47</td>
</tr>
<tr>
<td>recovery rate</td>
<td>43%</td>
<td>47%</td>
<td>39%</td>
<td>46%</td>
</tr>
</tbody>
</table>

First, observe that the behavior of $V$ is in keeping with the findings of Sections 3, 4 and 5. Moreover, notice that bondholders receive a maximum percentage of debt value upon liquidation under regime ND and that they are mostly guaranteed by tax plans allowing no tax refund in case of poor corporate performance. The recovery rate is the lowest under regime FD because of delayed closure which reduces the value of the firm. Note that, however, if the tax is flat the recovery rate is the same as in the no tax case. Indeed, in view of (3) the liquidation trigger $V$ remains stuck in $V_0^0$, which is the liquidation threshold in absence of taxation, if and only if $\tau_c^* = \tau_c$ and $x^* = x$, that is, only under a truly flat tax scheme. We conclude that only a flat tax plan does not interfere with liquidation, provided that the regime designs symmetric treatment of gain and losses like FD. In fact, a flat tax with a different regime would change the liquidation threshold. Supporters of flat tax plans must be aware of the crucial role of the different rules about deductibility. Another conclusion one can draw from this paper is that policy-makers can modulate progressivity, tax benefits, rules on deductibility in case of losses, to achieve their objectives, whether they aim at protecting bondholders or at delaying the closure of a firm, for example in presence of a reorganization plan for declining industries.

In the debate among tax policy specialists there is no consensus on the appropriate rate structure reforms that should be undertaken. Our paper provides a further contribution in this debate on the specific issue of corporate liquidation policies illustrating both the effects of a progressive vs proportional tax rate with different tax rules and providing precise policy prescriptions.

References

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